Paper / Subject Code: N10411 / Applied Mathematics-I

Applied Mathematics-I
Revised Course (NEP-2020)
(Dec 2024)

Marks-60
Durations-02 Hours

17-12-2024

Question 1 is compulsory.
Attempt any 3 from questions 2 to 6.
Scientific Calculator is allowed to use.

FE (All Branches)

1. Attempt any **Five** questions. **(Compulsory Problem)**
   - (a) Find all values of $(1+i)^{1/3}$. — 3
   - (b) Prove that: $tanh(log\sqrt{3}) = 0.5$ — 3
   - (c) If $u = (1-2xy+y^2)^{-1/2}$, prove that $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = y^2u^3$. — 3
   - (d) Find n^{th} derivative of $y = e^{ax}\sin^2 x$. — 3
   - (e) Show that the matrix $A = \frac{1}{\sqrt{3}}\begin{bmatrix} 1 & 1+i \\ 1-i & -1 \end{bmatrix}$ is unitary and hence find A^{-1}. — 3
   - (f) Using Newton Raphson's Method, find an iterative formula for $\sqrt[3]{150}$. — 3

2. (a) If $H = f(y-z, z-x, x-y)$, then prove that $\frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} + \frac{\partial H}{\partial z} = 0$. — 4
   (b) Prove that $\log(e^{i\alpha} + e^{i\beta}) = \log\left[2\cos\left(\frac{\alpha-\beta}{2}\right)\right] + i\left(\frac{\alpha+\beta}{2}\right)$. — 5
   (c) Apply the Gauss Seidel method, to solve the following system of linear equations up to the two iterations. — 6
   $15x + 3y - 2z = 85,\ x - 2y + 8z = 5,\ 2x + 10y + z = 51$.

3. (a) Discuss the maxima and minima of $(x^2 + y^2 + 8x + 6y + 6)$. — 4
   (b) Solve the equation $x^4 + x^3 + x^2 + x + 1 = 0$. — 5
   (c) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial x^2} = -\frac{\sin u \cos 2u}{4\cos^3 u}$. — 6

4. (a) Prove that $sech^{-1}(\sin\theta) = \log(\cot\frac{\theta}{2})$. — 4
   (b) Prove that $cos^6\theta = \frac{1}{32}(cos6\theta + 6cos4\theta + 15cos2\theta + 10)$. — 5
   (c) Find nonsingular matrices P & Q such that PAQ is in normal form and hence find rank of the matrix A for the following matrix $A = \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$. — 6

5. (a) Find a, b, c & A^{-1} if $A = \frac{1}{9}\begin{bmatrix} 8 & -4 & a \\ 1 & 4 & b \\ 4 & 7 & c \end{bmatrix}$ is orthogonal. — 4
   (b) State & prove Euler's theorem on homogeneous functions with two independent variables. — 5
   (c) If $y = e^{tan^{-1}x}$ then prove that $(1+x^2)y_{n+2} + [2(n+1)x - 1]y_{n+1} + n(n+1)y_n = 0$. — 6

6. (a) Find n^{th} derivative of $y = \frac{x}{1+3x+2x^2}$. — 4
   (b) Investigate for what values of λ and μ the system of linear equations $x + y + z = 6;\ x + 2y + 3z = 10;$ $x + 2y + \lambda z = \mu$ will have (i) No solution, (ii) Unique solution, (iii) Infinite number of solutions. — 5
   (c) Find the three sets of iterative solutions of the following equations by Gauss Jacobi method: — 6
   $5x - y + z = 10;\ 2x + 4y = 12;\ x + y + 5z = -1$.

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