RGIT
FE-SAHYOG

ENGINEERING MECHANICS Question Bank

📢 Important Note Regarding Engineering Mechanics Question Bank

Dear First-Year Engineering Students,

We are pleased to provide you with a carefully prepared Engineering Mechanics Question Bank based on the analysis of past year question papers (PYQs) adhering to the NEP 2020 syllabus.

Please read this **caution** carefully:

1. **Reference Material Only:** This Question Bank is solely for reference and practice purposes. It is designed to highlight the most frequently tested concepts and question structures from the past.
2. **No Guarantee:** There is absolutely no guarantee that the questions in the upcoming examination will be drawn *exclusively* from this set.
3. **Comprehensive Preparation is Key:** High scores are achieved by mastering the entire syllabus (Modules I through VI), understanding the underlying concepts, and practicing the theoretical and numerical questions.
4. **Do check PYQs for questions that are not given here**

Our recommendation is to **use this Question Bank as a guide to:**

- **Identify High-Priority Topics**
- **Practice solving Numericals** (3-mark, 6-mark and 7-mark questions).

Do not limit your study to these questions alone. Ensure you cover all the concepts listed in the Engineering Mechanics syllabus for complete and thorough preparation.

Read the question bank till the **Last Page** for maximum benefit.

Best of luck with your studies!

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List of IMP/Frequent Questions Module Wise:

Module I: System of Forces

1. **Resultant of Coplanar Force System (numerical)**
   Find the resultant force and locate its position w.r.t a point (e.g., Point O).
2. **Space Forces (numerical)**
   Find components of a force given coordinates or angles.
3. **Resolution of Forces (numerical)**
   Resolve a force into parallel components or find unknown forces for a specific resultant.
4. **Varignon's Theorem**

Module II: Centroid

1. **Centroid of Shaded Lamina (numerical)**
   Find the coordinate of the centroid for a shaded area (combination of triangle, semi-circle, rectangle).

Module III: Equilibrium of Force system and Friction

1. **Conditions of Equilibrium (Theory)**
2. **Equilibrium of Connected Bodies/Rollers (numerical)**
   Two spheres/rollers in a channel or inclined planes. Find reactions at contact points.
3. **Support Reactions of Beams (numerical)**
   Calculate support reactions (A and B) for a beam with UVL, UDL, and point loads.
4. **Laws of friction (theory)**
5. **Friction on Inclined Plane (numerical)**
   Determine force P required to prevent slipping or cause motion.

Module IV: Kinematics of particle and rigid bodies

1. **Curvilinear Motion / Variable Acceleration**
   Particle moving in X-Y plane or path equation given. Find velocity and acceleration.
2. **Instantaneous Center of Rotation (ICR)**
   Rod/Linkage mechanism. Find angular velocity or velocity of a point using ICR.

3. **Rectilinear motion**
4. **Difference between Curvilinear and Rectilinear motion.**

Module V: Kinetics of Particle

1. **Impulse Momentum / Variable Force (numerical)**
   Velocity of a crate/block subjected to variable force over time, or impact problems.
2. **D'Alembert's Principle / Connected Bodies**
   System of blocks connected by pulley/cables. Find acceleration and tension.
3. **Impact Definitions**
   Define/Explain Direct central impact and Oblique central impact.

Module VI: Introduction to Robot Kinematics

1. **Robotics Theory/Calculation**

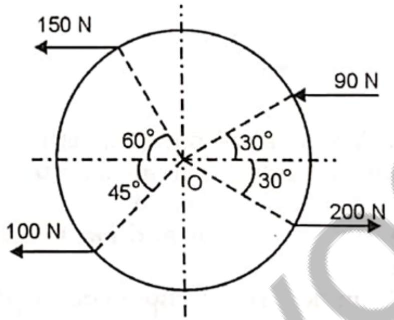
- Write D-H parameters and locate end-effector position.
- Classify Robot Mechanics and explain parts of a robotic arm.

Solutions

Module 1: System of Forces

1. Resultant of Coplanar Force System

Q. Determine the resultant of the coplanar parallel forces shown and locate it w.r.t. O.
Radius of circle = 2 m



Solution:

Given Data:
System: Coplanar Parallel Force System (All forces are horizontal).

Radius of circle (r): 2 m.

Forces & Geometry:

1. 150 N ($\leftarrow$) at 60° from horizontal (Top Left).
2. 90 N ($\leftarrow$) at 30° from horizontal (Top Right).
3. 200 N ($\rightarrow$) at 30° from horizontal (Bottom Right).
4. 100 N ($\leftarrow$) at 45° from horizontal (Bottom Left).

Step 1: Determine the Magnitude and Direction of Resultant (R)

Since all forces are horizontal, we simply sum them up algebraically.

$$R = \sum F_x$$

$$R = (+200) - 90 - 150 - 100$$

$$R = 200 - 340$$

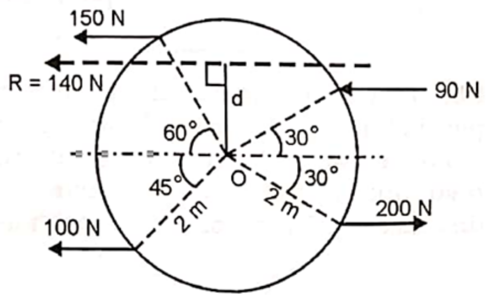
$$R = -140\text{ N}$$

Since the value is negative, the Resultant force is acting to the Left.

$$R = 140\ \text{N}(\leftarrow)$$

Step 2: Locate the Resultant using Varignon's Theorem

We need to find the perpendicular distance (d) of the resultant from the center O.



According to Varignon's Theorem:

The algebraic sum of moments of all forces about any point is equal to the moment of their resultant about the same point.

$$\sum M_O = M_O^R$$

A. Calculate sum of moments of all forces about Center O:

- **Moment of 150 N:** Force is left, acting above center.

$$M_1 = +(150) \times (2 \sin 60^\circ) = +259.81\ \text{Nm}$$

- **Moment of 90 N:** Force is left, acting above center.

$$M_2 = +(90) \times (2 \sin 30^\circ) = +90.00\ \text{Nm}$$

- **Moment of 200 N:** Force is right, acting below center.

$$M_3 = +(200) \times (2 \sin 30^\circ) = +200.00\ \text{Nm}$$

- **Moment of 100 N:** Force is left, acting below center.

$$M_4 = -(100) \times (2 \sin 45^\circ) = -141.42\ \text{Nm}$$

Total Moment:

$$\sum M_O = 259.81 + 90.00 + 200.00 - 141.42$$

$$\sum M_O = +408.39\ \text{Nm} \quad (\text{Counter-Clockwise})$$

B. Calculate position (d):

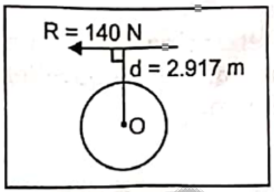
Moment of Resultant (R) about O must also be $+408.39$ Nm.

$$R \times d = \sum M_O$$

$$140 \times d = 408.39$$

$$d = \frac{408.39}{140}$$

$$d = 2.917 \text{ m}$$



Resultant Force

2. Space Forces

Q. The direction of a force is given $\theta_x = 66^\circ$ and $\theta_y = 140^\circ$. If $F_z = -4\text{ N}$, determine: i) θ_z ii) The magnitude of force (F) iii) The other components $(F_x$ and $F_y)$.

Solution:
Given Data:

- Direction angle with x-axis, $\theta_x = 66^\circ$
- Direction angle with y-axis, $\theta_y = 140^\circ$
- Z-component of force, $F_z = -4\text{ N}$

Concept / Formula Used:
For a force vector (F) in space, the direction cosines satisfy the relation:

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

The components are related to the magnitude (F) by:

$$F_x = F\cos\theta_x, \quad F_y = F\cos\theta_y, \quad F_z = F\cos\theta_z$$

Step 1: Determine the Angle θ_z
Using the direction cosine identity:

$$(0.4067)^2 + (-0.7660)^2 + \cos^2\theta_z = 1$$

$$0.1654 + 0.5868 + \cos^2\theta_z = 1$$

$$0.7522 + \cos^2\theta_z = 1$$

$$\cos^2\theta_z = 1 - 0.7522$$

$$\cos^2\theta_z = 0.2478$$

Taking the square root:

$$\cos\theta_z = \pm\sqrt{0.2478}$$

$$\cos\theta_z = \pm 0.4978$$

We are given that $F_z = -4\,\mathrm{N}$. Since $F_z = F\cos\theta_z$ and magnitude F is always positive, the sign of $\cos\theta_z$ must match the sign of F_z. Since F_z is negative, $\cos\theta_z$ must be **negative**.

$$\cos\theta_z = -0.4978$$

$$\theta_z = \cos^{-1}(-0.4978)$$
$$\theta_z = 119.85^\circ$$

Step 2: Determine the Magnitude of Force (F)
Using the z-component formula:

$$F_z = F\cos\theta_z$$

$$-4 = F \times (-0.4978)$$

$$F = \frac{-4}{-0.4978}$$

$$F = 8.035\,\mathrm{N}$$

Step 3: Determine the Other Components (F_x and F_y)
Calculate F_x:

$$F_x = F\cos\theta_x$$

$$F_x = 8.035 \times \cos(66^\circ)$$

$$F_x = 8.035 \times 0.4067$$

$$F_x = 3.268\,\mathrm{N}$$

Calculate F_y:

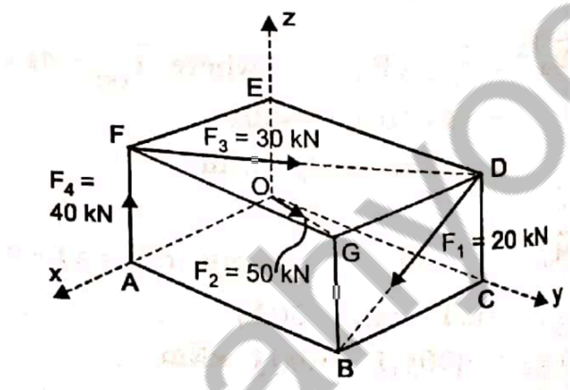
$$F_y = F\cos\theta_y$$

$$F_y = 8.035 \times \cos(140^\circ)$$

$$F_y = 8.035 \times (-0.7660)$$

$F_y = -6.155\ \mathrm{N}$

Q. Determine the resultant force and couple moment about the origin of the force system shown in figure.



Solution:
The given system is a general system of four forces. The coordinates of the various points through which the forces pass are:
A (4, 0, 0), B (4, 5, 0), C (0, 5, 0), D (0, 5, 3), F (4, 0, 3), G (4, 5, 3) and O (0, 0, 0).

Putting the forces in vector form

Force $\vec{F_1}$

$$\vec{F_1} = F_1\hat{e}_{DB}$$

$$\hat{e}_{DB} = \frac{4\mathbf{i} - 3\mathbf{k}}{\sqrt{4^2 + 3^2}}$$

$$\vec{F_1} = 20\left(\frac{4\mathbf{i} - 3\mathbf{k}}{5}\right)$$

$$\boxed{\vec{F_1} = 16\mathbf{i} - 12\mathbf{k}\ \mathrm{kN}}$$

Force $\vec{F}_2$

$$\vec{F}_2 = F_2 \hat{e}_{OG}$$

$$\hat{e}_{OG} = \frac{4\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}}{\sqrt{4^2 + 5^2 + 3^2}}$$

$$\vec{F}_2 = 50\left(\frac{4\mathbf{i} + 5\mathbf{j} + 3\mathbf{k}}{\sqrt{50}}\right)$$

$$\boxed{\vec{F}_2 = 28.28\mathbf{i} + 35.35\mathbf{j} + 21.21\mathbf{k}\ \mathrm{kN}}$$

Force $\vec{F}_3$

$$\vec{F}_3 = F_3 \hat{e}_{FD}$$

$$\hat{e}_{FD} = \frac{-4\mathbf{i} + 5\mathbf{j}}{\sqrt{4^2 + 5^2}}$$

$$\vec{F}_3 = 30\left(\frac{-4\mathbf{i} + 5\mathbf{j}}{\sqrt{41}}\right)$$

$$\boxed{\vec{F}_3 = -18.74\mathbf{i} + 23.42\mathbf{j}\ \mathrm{kN}}$$

Force $\vec{F}_4$

$$\vec{F}_4 = 40\mathbf{k}\ \mathrm{kN}$$

Resultant Force

$$\vec{R} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

$$\vec{R} = (16\mathbf{i} - 12\mathbf{k}) + (28.28\mathbf{i} + 35.35\mathbf{j} + 21.21\mathbf{k}) + (-18.74\mathbf{i} + 23.42\mathbf{j}) + (40\mathbf{k})$$

$$\boxed{\vec{R} = 25.54\mathbf{i} + 58.77\mathbf{j} + 49.21\mathbf{k}\ \mathrm{kN}}$$

Moment of Forces about Origin

Moment due to $\vec{F}_1$

$$\vec{M}_{O1} = \vec{r}_{OD} \times \vec{F}_1 \quad \text{where } \vec{r}_{OD} = 5\mathbf{j} + 3\mathbf{k}$$

$$\vec{M}_{O1} = (5\mathbf{j} + 3\mathbf{k}) \times (16\mathbf{i} - 12\mathbf{k})$$

$$\boxed{\vec{M}_{O1} = -60\mathbf{i} + 48\mathbf{j} - 80\mathbf{k}\ \mathrm{kNm}}$$

Moment due to $\vec{F}_2$

$$\vec{M}_{O2} = 0 \quad (\text{since } \vec{F}_2 \text{ passes through origin})$$

Moment due to $\vec{F}_3$

$$\vec{M}_{O3} = \vec{r}_{OD} \times \vec{F}_3$$

$$\vec{M}_{O3} = (5\mathbf{j} + 3\mathbf{k}) \times (-18.74\mathbf{i} + 23.42\mathbf{j})$$

$$\boxed{\vec{M}_{O3} = -70.26\mathbf{i} - 56.22\mathbf{j} + 93.7\mathbf{k} \text{ kNm}}$$

Moment due to $\vec{F}_4$

$$\vec{M}_{O4} = \vec{r}_{OA} \times \vec{F}_4 \quad \text{where } \vec{r}_{OA} = 4\mathbf{i}$$

$$\vec{M}_{O4} = (4\mathbf{i}) \times (40\mathbf{k})$$

$$\boxed{\vec{M}_{O4} = -160\mathbf{j} \text{ kNm}}$$

Resultant Moment at Origin

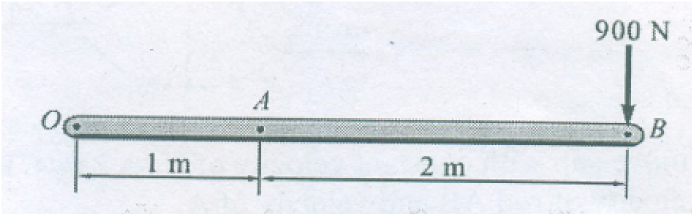
$$\vec{M}_O = \vec{M}_{O1} + \vec{M}_{O2} + \vec{M}_{O3} + \vec{M}_{O4}$$

$$\vec{M}_O = (-60\mathbf{i} + 48\mathbf{j} - 80\mathbf{k}) + 0 + (-70.26\mathbf{i} - 56.22\mathbf{j} + 93.7\mathbf{k}) + (-160\mathbf{j})$$

$$\boxed{\vec{M}_O = -130.26\mathbf{i} - 168.2\mathbf{j} + 13.7\mathbf{k} \text{ kNm}}$$

3. Resolution of Forces

Resolve the Force F = 900 N acting at B as shown in Fig. into parallel components at O and A.



Solution:

Given:

Force acting at B,

F = 900 N (downward)

Distance from O to B,

OB = OA + AB = 1 + 2 = 3 m

Step 1: Moment of the force about point O

Moment of a force about a point is given by:

M_O = F x m

M_O = 900 x 3

M_O = 2700 N.m

Since the force causes clockwise rotation about point O,

M_O = 2700 N.m (clockwise)

Step 2: Equivalent Force–Couple System at O

A force acting at a point can be transferred to another point by:

• Applying the same force at the new point

• Adding a couple equal to the moment produced

Equivalent system at O:

• Force = 900 N downward acting at O

• Couple = 2700 N·m (clockwise)

4. Varignon's Theorem

Q.State and prove Varignon's Theorem:

Varignon, a French mathematician (1654 – 1722) established that the sum of the moments of a concurrent system of forces about any point is equal to the moment of the resultant of the concurrent system about the same point. Though originally derived for a concurrent system of forces, this theorem can in fact be applied to any system of forces and is thus stated as "*the algebraic sum of the moments of a system of coplanar forces about any point in the plane is equal to the moment of the resultant force of the system about the same point*".
Mathematically it is written as

$$\Sigma M_A^F = M_A^R \quad \text{............ 2.2}$$

Sum of moments of all forces about any point, say point A. = Moment of the resultant about the same point A.

Proof – Let P and Q be two concurrent forces at O, making angle θ_1 and θ_2 with the x-axis, let R be their resultant making an angle θ with x-axis.
Let A be a point on the y-axis about which we shall find the moments of P and Q and also of the resultant R. Let d_1, d_2 and d be the moment arm of P, Q and R from moment centre A.

Let the x component of forces P, Q and R be P_x , Q_x and R_x respectively

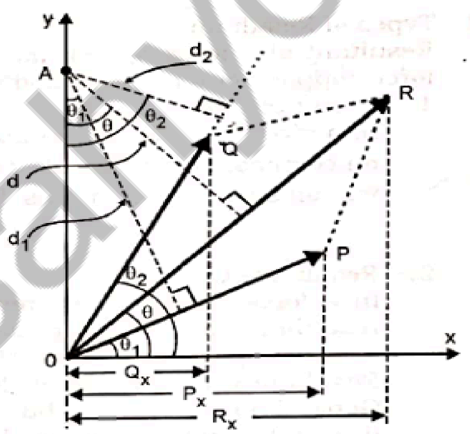


Fig. 2.12

Now, Moment of P about A $= M_A^P = P \times d_1$ --------(1)
Moment of Q about A $= M_A^Q = Q \times d_2$ --------(2)
Moment of R about A $= M_A^R = R \times d$
$= R\,(\,OA \cos\theta\,)$
$= OA\,(\,R_x\,)$ --------(3)

Adding equations (1) and (2) we have

$M_A^P + M_A^Q = P\,d_1 + Q\,d_2$

or sum of moments $\Sigma M_A^F = +\,(\,P \times OA \cos\theta_1\,) + (\,Q \times OA \cos\theta_2\,)$

$= OA.\,P_x + OA.Q_x$ since $P_x = P\cos\theta_1$ and $Q_x = Q\cos\theta_2$

$= OA\,(\,P_x + Q_x\,)$

$\therefore \quad \Sigma M_A^F = OA\,(\,R_x\,)$ ---------(4)

$P_x + Q_x = R_x$ since the resultant of forces in the 'x' direction is the sum of components of forces in the 'x' direction

Comparing equation (4) with (3)

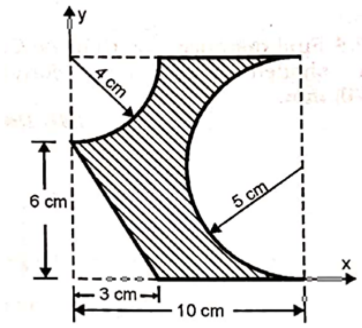
$\Sigma M_A^F = M_A^R$ ----- Proved

The above equation can similarly be extended for more than two forces in the system.

Module 2: Centroid

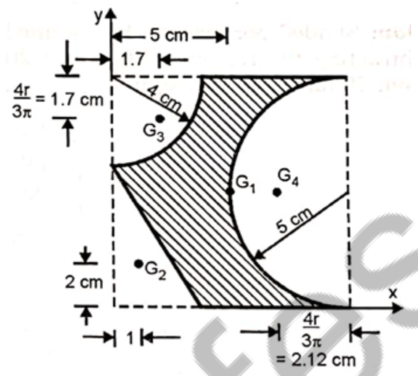
1. Centroid of Shaded Lamina

Q. Find centroid of shaded area shown



Solution:

The shaded area can be obtained by taking an entire square of
10 cm x 10 cm and subtracting a quarter-circle, a triangle and a semicircle.

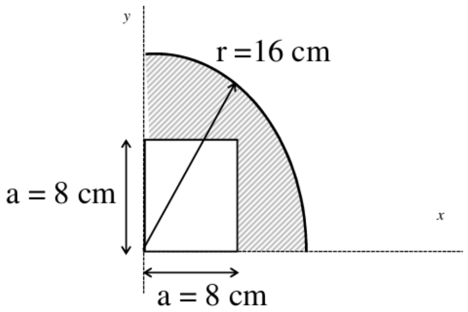


Part	Area (A_i) cm²	x_i cm	y_i cm	A_ix_i cm³	A_iy_i cm³
1. Square	100	5	5	500	500
2. Rt. Triangle	- 9	1	2	- 9	- 18
3. Quarter-circle	- 12.57	1.697	8.302	- 21.32	- 104.33
4. Semi-circle	- 39.27	7.878	5	- 309.37	- 196.35
	ΣA_i = 39.16			ΣA_ix_i = 160.31	ΣA_iy_i = 181.32

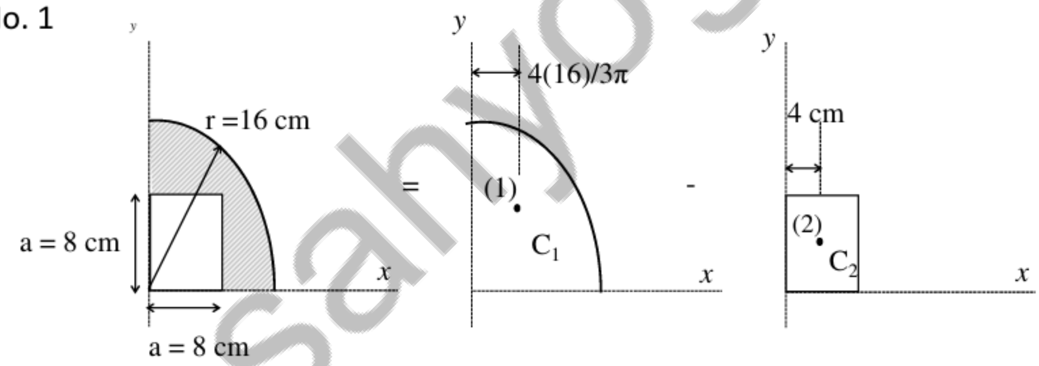
$$\bar{X} = \frac{\Sigma A_i x_i}{\Sigma A_i} = \frac{160.31}{39.16} = 4.09 \text{ cm} \quad \text{and} \quad \bar{Y} = \frac{\Sigma A_i y_i}{\Sigma A_i} = \frac{181.32}{39.16} = 4.63 \text{ cm}$$

$$\therefore \bar{X}, \bar{Y} = (4.09, 4.63) \text{ cm} \quad \text{......... Ans.}$$

Q. Locate the centroid of the plane shaded area shown below.



Solution of Q. No. 1



	A (cm²)	$\bar{x}$, cm	$\bar{x}$A, cm³
1	$\frac{\pi(16^2)}{4}$ = 201.06	$\frac{4(16)}{3\pi}$ = 6.7906	1365.32
2	-(8)(8) = - 64	4	-256
Σ	137.06		1109.32

Then $\bar{X} = \frac{\sum \bar{x}A}{\sum A} = \frac{1109.32}{137.06} = 8.09$ cm

$\bar{Y} = 8.09$ cm ($\bar{Y} = \bar{X}$ by symmetry)

Module 3: Equilibrium of Force system and Friction

1. Conditions of Equilibrium

Solution:

Conditions of Equilibrium (COE)

A body is said to be in equilibrium if it is in a state of rest or uniform motion. This is precisely what Newton has stated in his first law of motion. For a body to be in equilibrium the resultant of the system should be zero. This implies that:

the sum of all forces should be zero i.e. $\sum \overline{F} = 0$

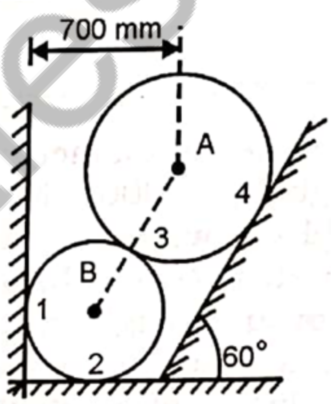
and the sum of all moments should also be zero i.e. $\sum \overline{M} = 0$

The above two equations are the conditions of equilibrium in vector form.
For a coplanar system of forces, the scalar equations of equilibrium are:

$\sum F_x = 0$ ------------------- sum of all forces in x direction is zero
$\sum F_y = 0$ ------------------- sum of all forces in y direction is zero
$\sum M = 0$ ------------------- sum of moments of all forces is zero

2. Equilibrium of Connected Bodies/Rollers

Q. Two spheres **A** and **B** of weight **1000 N** and **750 N** respectively are kept as shown in the figure. Determine the reactions at all contact points 1, 2, 3, and 4. Radius of sphere **A = 400 mm** and radius of sphere **B = 300 mm**.



Solution:

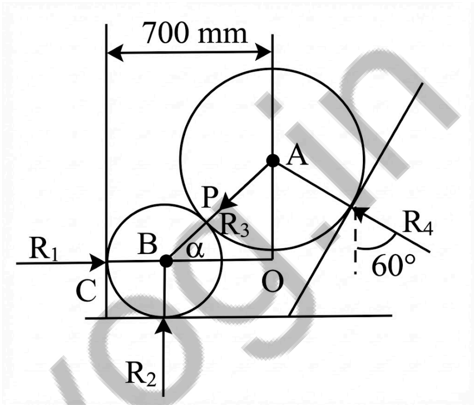
Given:

- Weight of sphere A = 1000 N
- Weight of sphere B = 750 N
- Radius of sphere A = 400 mm

- Radius of sphere B = 300 mm
- The contact points are labelled as 1, 2, 3, and 4 in the diagram.

Find: Reactions at contacts points 1, 2, 3, and 4.

BC=BP=300mm=0.3m
AP=400 mm=0.4 m
AB = AP + BP
= 0.7m
CO = BC + BO
0.7 = 0.3 + BO
BO = 0.4m



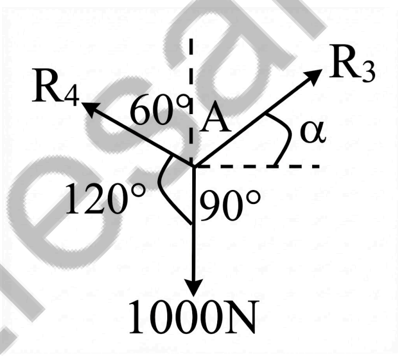
In △AOB

$$\cos \alpha = \frac{BO}{AB} = \frac{0.4}{0.7}$$

$$\alpha = \cos^{-1}(0.5714)$$

$$\boldsymbol{\alpha = 55.1501^o}$$

Forces R3, R4 and 1000N are under equilibrium at point A

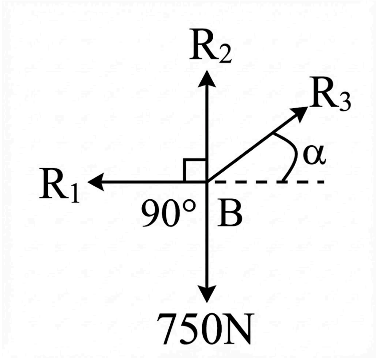


Applying Lami's Theorem,

$$\frac{R3}{\sin 120} = \frac{1000}{\sin(150 - \alpha)} = \frac{R4}{\sin(90 + \alpha)}$$

$$\frac{R3}{\sin 120} = \frac{1000}{\sin(150 - 55.1501)} = \frac{R4}{\sin(90 + 55.1501)}$$

Solving the equations
R3= 869.1373 N
R4= 573.4819 N



Forces R1,R2,R3 and 750N are under equilibrium at B

Applying conditions of equilibrium
ΣFY=0
-R3 sin α-750+R2=0
R2 = 869.1373 sin(55.1501)+750
R2 = 1463.2591 N (Acting upwards)

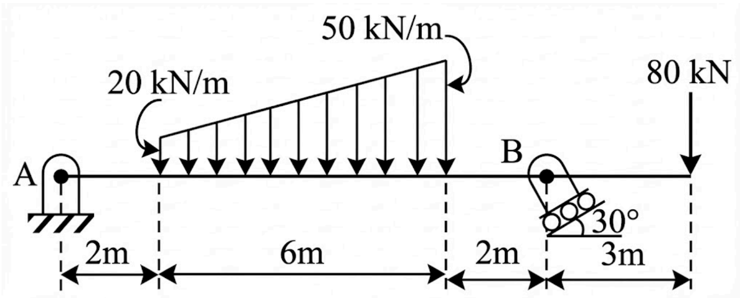
Applying conditions of equilibrium
ΣFx=0
R1 - R3 cos α = 0

R1= 869.1373 cos (55.1501)
R1= 496.65 N (Acting towards right)

Point	**Force**
R_1	**496.65 N(Towards right)**
R_2	**1463.2591 N(Towards up)**
R_3	**869.1373 N(55.1501^0 in first quadrant)**
R_4	**573.4819 N(30^0 in second quadrant)**

3. Support Reactions of Beams

Q. Find the support reactions at A and B for the beam loaded as shown in the given figure.



Solution:

Given: Various forces on beam

To find: Support reactions at A and B

Solution: Draw PQ L to RS

Effective force of uniform load = 20 x 6 = 120 kN

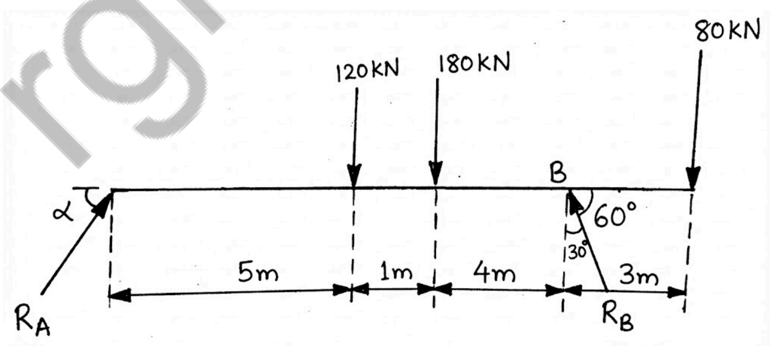
$$2 + \frac{6}{2} = 5 \text{ m}$$

This load acts at 5m from A

Effective force of uniformly varying load $= \frac{1}{2} \times (80\text{-}20) \times 6$ =180kN

$$2 + \frac{6}{3} \times 2 = 6\text{m}$$

This load acts at 6m from A



The beam is in equilibrium
Applying the conditions of equilibrium

$\Sigma MA = 0$

$-120 \times 5 - 180 \times 6 + RB\cos 30 \times 10 - 80 \times 13 = 0$

$10RB\cos 30 = 120 \times 5 + 180 \times 6 + 80 \times 13$

$RB = 314.0785$ N

Reaction at B will be at 60o in second quadrant

$\Sigma Fx = 0$

$R_A\cos\alpha - RB\sin 30 = 0$

$R_A\cos\alpha - 314.0785 \times 0.5 = 0$

$R_A\cos\alpha = 157.0393$ N ……..(1)

$\Sigma Fy=0$

$R_A\sin\alpha - 120 - 180 + RB\cos 30 - 80 = 0$

$R_A\sin\alpha = 12 + 180 - 314.0785 \times 0.866 + 80$

$R_A\sin\alpha = 108.008N$ ……..(2)

Squaring and adding (1) and (2)

$R_A{}^2(\sin^2\alpha + \cos^2\alpha) = 36325.3333$

$R_A = 190.5921$ N

Dividing (2) by (1)

$$\frac{R_A \sin\alpha}{R_A \cos\alpha} = \frac{108.008}{157.0393}$$

$\alpha=\tan^{-1}(0.6877)$

= 34.5173

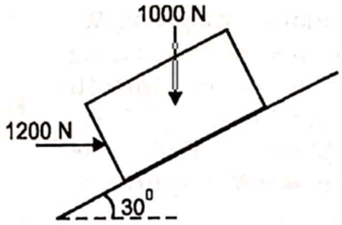
4. Laws of friction

Solution:

1. The frictional force is always tangential to the contact surface and acts opposite to the direction of impending motion.
2. The value of frictional force F increases as the applied disturbing force increases till it reaches the limiting value F_{max}. At this limiting stage, the body is on the verge of motion.
3. The ratio of limiting frictional force F_{max} and the normal reaction N is a constant and it is referred to as the **coefficient of static friction** (μ_s).
4. For bodies in motion, the frictional force developed (F_k) is less than the limiting frictional force (F_{max}). The ratio of F_k and the normal reaction N is a constant and is referred to as the **coefficient of kinetic friction** (μ_k).
5. The frictional force F generated between the two rubbing surfaces is independent of the area of contact.

5. Friction on Inclined Plane

Q. If a horizontal force of 1200 N is applied to a block of 1000 N, then will the block be held in equilibrium or slide down or move up?
Take μ = 0.3.



Solution: This problem is different from the previous problems, since we are required to find out the state of the block i.e. whether it is in equilibrium or not. If not, in which direction it is moving.

Let F be the friction force acting down the plane be required to keep the block in equilibrium. Here we cannot take $F = \mu N$ because the block may not be on the verge of motion.

Taking the axes as shown

$\Sigma F_y = 0$

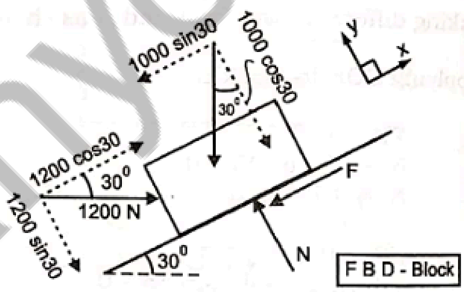
$N - 1000 \cos 30 - 1200 \sin 30 = 0$

∴ $N = 1466$ N

$\Sigma F_x = 0$

$1200 \cos 30 - 1000 \sin 30 - F = 0$

∴ $F = 539.2$ .... $F_{required}$



∴ F = 539.2 N force is required to act down the plane to keep the block in equilibrium.

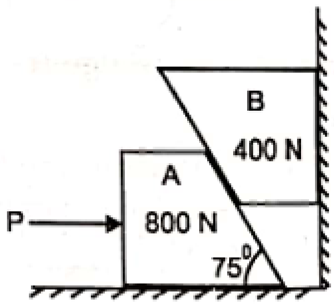
Now the maximum friction force the contact surface can produce

$= \mu N = 0.3 \times 1466$

$= 439.8$ N .... $F_{available}$

Since $F_{required} > F_{available}$ the block is not in equilibrium, but is moving up since F is directed down.

Q. Wedges A and B are held in equilibrium by the application of horizontal force P as shown. Find the minimum force P required to do so. Take $\mu = 0.2$ between the wedges, $\mu = 0.2$ between wedge B and wall and $\mu = 0.3$ wedge A and the floor.



Solution:

Imagine the given system when P = 0. In such a case the wedge B would slip down and wedge A would therefore slide to the left. For this not to happen, a minimum force P can prevent this and maintain equilibrium of the system.

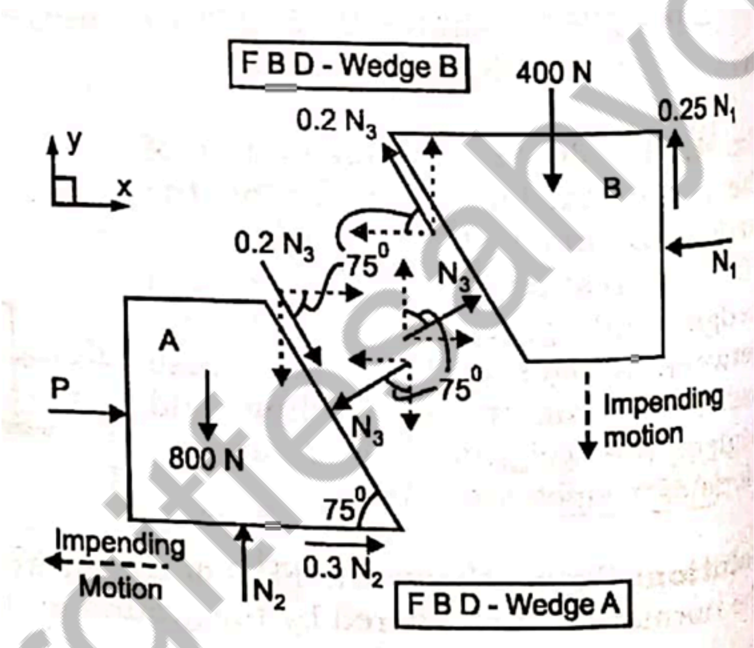
Isolating the wedges as shown

Applying COE to Wedge B

$\Sigma F_X = 0 \; -N_1 - 0.2N_3 \cos 75 + N_3 \sin 75 = 0 \therefore -N_1 + 0.914N_3 = 0$ ....(1)

$\Sigma F_Y = 0 \; 0.25N_1 + 0.2N_3 \sin 75 + N_3 \cos 75 - 400 = 0 \therefore 0.25N_1 + 0.452N_3 = 400$ .... (2)

Solving equations (1) and (2), we get, $N_1 = 537.25\ \mathrm{N}$ and $N_3 = 587.8\ \mathrm{N}$



Applying COE to Wedge A

$$\Sigma F_Y = 0 \; N_2 - 0.2N_3 \sin 75 - N_3 \cos 75 - 800 = 0$$

Substituting

$N_3 = 587.8\ \mathrm{N}$, we get $N_2 = 1065.7\ \mathrm{N}$

$$\Sigma F_X = 0 \; P + 0.3N_2 + 0.2N_3 \cos 75 - N_3 \sin 75 = 0$$

Substituting $N_2 = 1065.7\ \mathrm{N}$ and $N_3 = 587.8\ \mathrm{N}$, we get , $P = 217.6\ \mathrm{N}$

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Module 4: Kinematics of particle and rigid bodies

1. Curvilinear Motion / Variable Acceleration

Q. A particle starts from rest from origin and it's acceleration is given by $a = \frac{k}{(x+4)^2}$ m/s^2 Knowing that v = 4m/s when x=8 m, find: 1) Value of k 2) Position when v=4.5 m/s

Solution:

Particle starts from rest

$$a = \frac{k}{(x+4)^2} \quad m/s^2$$

We know that $a = v\frac{dv}{dx}$

$$v\frac{dv}{dx} = \frac{k}{(x+4)^2}$$

$$v\,dv = k(x+4)^{-2}\,dx$$

Integrating both sides

$$\int v dv = \int k(x+4)^{-2}\,dx$$

$$\frac{v^2}{2} = \frac{-k}{x+4} + c1 \ldots\ldots\ldots(1)$$

Putting x=0 and v=0

$$c1 = \frac{k}{4} \quad \ldots\ldots\ldots.(2)$$

$$\frac{v^2}{2} = \frac{-k}{x+4} + \frac{k}{4} \quad \ldots\ldots.\text{(From 1 and 2)} \ldots\ldots..(3)$$

k = 48

From (3)

$$\frac{v^2}{2} = \frac{-48}{x+4} + \frac{48}{4}$$

$v^2 = 24 - \dfrac{96}{x+4}$

Substituting v=4.5 m/s

$4.5^2 = 24 - \dfrac{96}{x+4}$

$\dfrac{96}{3.75} = x + 4$

x = 21.6 m

Value of k = 48

The particle is at a distance of 21.6 m from origin when v = 4.5 m/ s

Q. The curvilinear motion of a particle is defined by $v_x = 25 - 8t\ \mathrm{m/s}$ and $y = 48 - 3t^2$. Knowing at $t = 0, x = 0$, find at time $t = 4$ sec, the position, velocity and acceleration vectors. Also find the corresponding magnitudes.

Solution:

For the given curvilinear moving particle, working in rectangular system.

X-direction

Given $v_x = 25 - 8t$ .... (1)

using $v_x = \frac{dx}{dt} = 25 - 8t\ dx = (25 - 8t)dt$

Integrating taking lower limits as $x = 0$ and $t = 0$

$$\int_0^x dx = \int_0^t (25 - 8t)dt$$

$$x = 25t - 4t^2............(2)$$

also $a_x = \frac{dv_x}{dt}$

$$\therefore a_x = -8\ \mathrm{m/s^2}............(3)$$

Y-direction

$$y = 48 - 3t^2 ....(a)$$

using $v_y = \frac{dy}{dt}$ using $a_y = \frac{dv_y}{dt}$

$$v_y = -6t \text{ m/s}.........................(b)$$

$$a_y = -6 \text{ m/s}^2.........................(c)$$

To find position, velocity and acceleration at $t = 4$ sec

Substituting $t = 4$ in equation (2) and equation (a) we get

$$x = 36 \text{ m}, y = 0$$

$$\therefore r = 36\mathbf{i} + 0\mathbf{j} \text{ m} \quad ....\text{Ans.}$$

$$r = 36 \text{ m}$$

Substituting $t = 4$ in equation (1) and equation (b) we get,

$$v_x = -7 \text{ m/s}, \quad v_y = -24 \text{ m/s}$$

$$\therefore v = -7\mathbf{i} - 24\mathbf{j} \text{ m/s} \quad ......\text{Ans.}$$

$$v = 25 \text{ m/s}$$

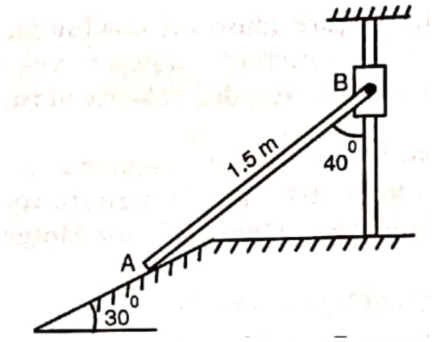
From equations (3) and (c), we find that the acceleration is constant in x and y direction

$$\therefore a = -8\mathbf{i} - 6\mathbf{j} \text{ m/s}^2 \quad .....\text{Ans.}$$

$$a = 10 \text{ m/s}^2$$

2. Instantaneous Center of Rotation (ICR)

Q. Figure shows a collar B which moves up with constant velocity of 2 m/s. To the collar is pinned a rod AB, the end A of which slides freely against a 30° sloping ground. For this instant, determine the angular velocity of the rod and velocity of end A of the rod.



Solution:
The system consists of two bodies in motion. Rod AB performs General Plane Motion and collar B performs Rectilinear Translation Motion.

General Plane Motion of rod AB

Let us use Instantaneous Centre Method.

(1) Velocity of point B on the G.P. body = the velocity of collar = 2 m/s. Also direction of velocity (Dov_B) is vertical.
Direction of velocity (Dov_A) end A is along the inclined plane.

(2) Drawing the perpendiculars to the direction of velocity at A and B. Let the point of intersection be I.

(3) From geometry the radial lengths r_{AI} and r_{BI} need to be worked out.

Using sine rule to solve Δ ABI

$$\frac{1.5}{\sin 60} = \frac{r_{AI}}{\sin 50} = \frac{r_{BI}}{\sin 70}$$

$$\therefore \quad r_{AI} = 1.327 \text{ m} \quad \text{and} \quad r_{BI} = 1.627 \text{ m}$$

4. Since I is the instantaneous centre of rotation of the G.P body AB, we have

$$v_B = r_{BI} \times \omega_{AB}$$

$$\therefore \quad \omega_{AB} = 1.229 \text{ rad/s} \circlearrowleft \quad \text{.........}\textbf{Ans.}$$

also

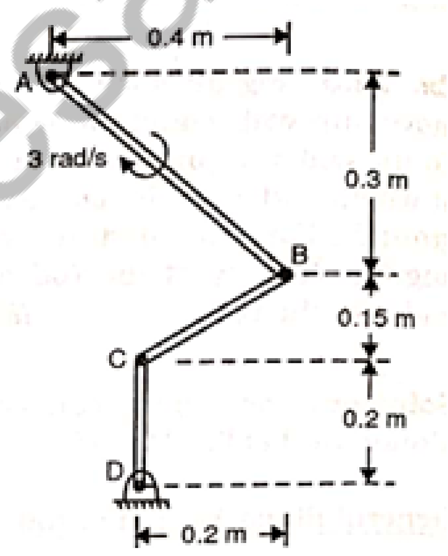
$$v_A = r_{AI} \times \omega_{AB}$$

$$= 1.327 \times 1.229$$

$$\therefore \quad v_A = 1.63 \text{ m/s}$$

$$\therefore \quad v_A = 1.63 \text{ m/s } \theta = 30^\circ \quad \text{.........}\textbf{Ans.}$$

Q. Figure shows a mechanism in motion. Rod AB has a constant angular velocity of 3 rad/s clockwise. Find angular velocity of rod BC and rod CD.



Solution:
The system consists of three bodies in motion. Rods AB and CD perform rotation motion and rod BC performs General Plane Motion.

Rotation Motion of rod AB

Rod AB rotates about A $\therefore$ velocity of end B $= v_B = r_{BA} \times \omega_{AB} = 0.5 \times 3 \therefore \quad v_B = 1.5 \text{ m/s}$

also the direction of velocity of v_B is $\perp$ to radial length AB.

General Plane Motion of rod BC

Let us use Instantaneous Centre Method.

1. Velocity of end B, $v_B = 1.5 \text{ m/s}$ and Dov_B is $\perp$ to radial length AB. Also direction of velocity of end C i.e. Dov_C is $\perp$ to radial length CD since C is also common to rod CD, which rotates about D.
2. Drawing the perpendiculars to the directions of velocity, Dov_B and Dov_C. Let the point of intersection be I.
3. From geometry the radial lengths r_{BI} and r_{CI} need to be worked out. Solving Δ BCI, $L(BC) = 0.25 \text{ m}$ Since Δ BCI is isosceles, $r_{BI} = L(BC) = 0.25 \text{ m}$ also $r_{CI} = 0.3 \text{ m}$
4. Since I is the instantaneous centre of rotation of the G.P body BC, we have $v_B = r_{BI} \times \omega_{BC} \therefore \quad 1.5 = 0.25 \times \omega_{BC}$ or $\omega_{BC} = 6 \text{ rad/s} \circlearrowleft \quad \text{.........}\mathbf{Ans.}$

also $v_C = r_{CI} \times \omega_{BC} = 0.3 \times 6 \therefore \quad v_C = 1.8 \text{ m/s} \leftarrow$

Rotation Motion of rod CD

Rod CD rotates about D Knowing C is a common end to both rods BC and rod CD, we use $v_C = 1.8 \text{ m/s}$ from above $v_C = r_{CD} \times \omega_{CD} \therefore \quad 1.8 = 0.2 \times \omega_{CD}$ or $\omega_{CD} = 9 \text{ rad/s} \circlearrowleft \quad \text{.........}\mathbf{Ans.}$

3. Rectilinear motion

Q. The car moves in a straight line such that for a short time its velocity is defined by $v = (9t^2 + 2t)\,\mathrm{m/s}$ where t is in seconds. Determine the position and acceleration when t = 3 sec.

Solution:

Given:

- Velocity equation: $v = (9t^2 + 2t)\,\mathrm{m/s}$
- Time: $t = 3\,\mathrm{sec}$
- Assumption: Initial position $s = 0$ at $t = 0$ (starting from origin).

1. Determine Acceleration (a)

Acceleration is the rate of change of velocity with respect to time (derivative of velocity).

$$a = \frac{dv}{dt}$$

$$a = \frac{d}{dt}(9t^2 + 2t)$$

$$a = 18t + 2$$

At $t = 3$ seconds:

$$a = 18(3) + 2$$

$$a = 54 + 2$$

$$a = 56\,\mathrm{m/s^2}$$

2. Determine Position (s)

Position is found by integrating velocity with respect to time.

$$s = \int v\,dt$$

$$s = \int (9t^2 + 2t)\,dt$$

$$s = \frac{9t^3}{3} + \frac{2t^2}{2} + C$$

$$s = 3t^3 + t^2 + C$$

Since we assume the car starts from the origin ($s = 0$ when $t = 0$), the constant $C = 0$.

At $t = 3$ seconds:

$$s = 3(3)^3 + (3)^2$$

$$s = 3(27) + 9$$

$$s = 81 + 9$$

$$s = 90\,\mathrm{m}$$

Q. The acceleration of a particle in rectilinear motion is given by .
Knowing that $a = 100 - 3x^2\,\mathrm{m/s^2}$ at t = 0, v = 0 and x = 0, find:

- (a) At what position the velocity is zero.
- (b) The velocity at x = 8 m.
- (c) At what position the particle acquires maximum velocity.

Solution:

Acceleration as a function of position:

$$a = f(x) = 100 - 3x^2\,\mathrm{m/s^2} \quad \ldots(1)$$

To find the velocity-position relationship (v-x), we use the chain rule for acceleration:

$$a = v\frac{dv}{dx}$$

Substituting equation (1):

$$v\frac{dv}{dx} = 100 - 3x^2$$

$$v\,dv = (100 - 3x^2)\,dx$$

Integrating both sides: Using the lower limits v = 0 and x = 0 (given in the problem):

$$\int_0^v v\,dv = \int_0^x (100 - 3x^2)\,dx$$

$$\left[\frac{v^2}{2}\right]_0^v = \left[100x - \frac{3x^3}{3}\right]_0^x$$

$$\frac{v^2}{2} = 100x - x^3$$

Multiplying by 2 to isolate v^2:

$$v^2 = 200x - 2x^3 \quad \ldots(2)$$

(a) At what position the velocity is zero

To find the position at zero velocity, substitute $v = 0$ into equation (2):

$$0 = 200x - 2x^3$$

$$0 = 2x(100 - x^2)$$

This gives solutions:

$$x = 0$$

$$100 - x^2 = 0 \implies x^2 = 100 \implies x = \pm 10$$

Answer: The particle acquires zero velocity at three positions:

$$x = 0, \quad x = +10\,\text{m}, \quad x = -10\,\text{m}$$

(b) The velocity at x = 8 m
Substitute x = 8 m into equation (2):

$$v^2 = 200(8) - 2(8)^3$$

$$v^2 = 1600 - 2(512)$$

$$v^2 = 1600 - 1024$$

$$v^2 = 576$$

$$v = \pm\sqrt{576}$$

(c) At what position the particle acquires maximum velocity
For maximum or minimum velocity, the acceleration (the derivative of velocity) becomes zero.

$$100 - 3x^2 = 0$$

$$3x^2 = 100$$

$$x^2 = \frac{100}{3}$$

$$x = \pm\sqrt{33.33}$$

$$x = \pm 5.774\,\text{m}$$

Determine the maximum velocity by substituting these x values into equation (2):

Case 1: Put $x = +5.774\,\mathrm{m}$

$$v^2 = 200(5.774) - 2(5.774)^3$$

Solving this yields:

$$v = 27.74\,\mathrm{m/s}$$

Case 2: Put $x = -5.774\,\mathrm{m}$ Substituting a negative x into equation (2) results in a negative value for v^2, which would give a complex number (imaginary velocity). Therefore, this position is not valid for real motion.

Velocity is maximum at x = +5.774 m and the maximum velocity is vmax = 27.74 m/s

4. Difference between Curvilinear and Rectilinear motion.

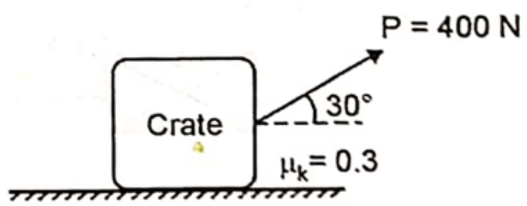
Solution:

Feature	Rectilinear Motion	Curvilinear Motion
1. Path of motion	Motion takes place along a straight line	Motion takes place along a curved path
2. Direction of motion	Direction of motion remains constant	Direction of motion continuously changes
3. Velocity direction	Velocity acts in a fixed direction	Velocity direction changes at every point
4. Acceleration	Acceleration may be zero or along the same straight line	Acceleration is always present due to change in direction
5. Dimensional nature	It is a one-dimensional motion	It is a two-dimensional motion
6. Examples	Elevator motion, free-fall, train on straight track	Circular motion, projectile (parabolic) motion

Module 5: Kinetics of Particle

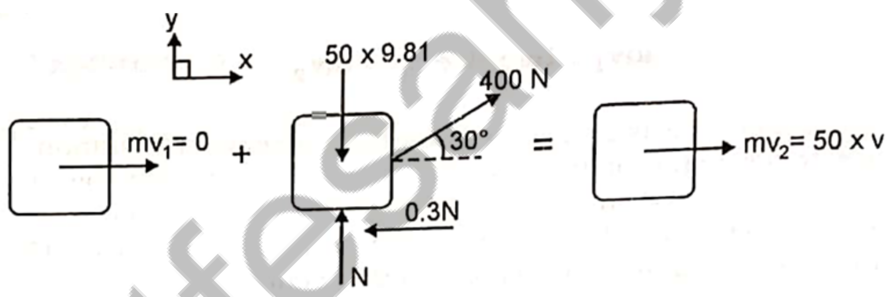
1. Impulse Momentum / Variable Force

Q. The 50 kg crate shown in figure, rest on horizontal plane for which the coefficient of kinetic friction is $\mu_k = 0.3$. If the crate does not tip over when it is subjected to a 400 N force, determine the velocity of the crate in 8 sec starting from rest.



Solution:

Applying Impulse Momentum Equation to the 50 kg blocks during its 8 sec motion period.



Applying Impulse Momentum Equation in the x direction

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$0 + [400\cos 30 - 0.3 \times 290.5] \times 8 = 50v$$

$$\therefore \quad v = 41.48 \text{ m/s} \quad \text{.........}\mathbf{Ans.}$$

Applying Impulse Momentum Equation in the y direction

$$mv_1 + \text{Impulse}_{1-2} = mv_2$$

$$0 + [400 \sin 30 - 50 \times 9.81 + N] \times 8 = 0$$

$$\therefore \quad N = 290.5\text{ N}$$

2. D'Alembert's Principle / Connected Bodies

Q. A block of mass 5 kg is released from rest along a 40 degree inclined plane. Determine the acceleration of the block using D' Alembert's principle. Take coefficient of friction.

Solution:

The FBD (Free Body Diagram) of the block in dynamic equilibrium is shown (implied in text). Applying D' Alembert's principle.

Using $\sum F_y = 0$ (Perpendicular to the plane):

$$N - 49.05 \cos 40^\circ = 0$$

(Note: 49.05 represents the weight $mg = 5 \times 9.81$)

$$\therefore N = 37.57\,\text{N}$$

Using $\sum F_x = 0$ (Along the plane):

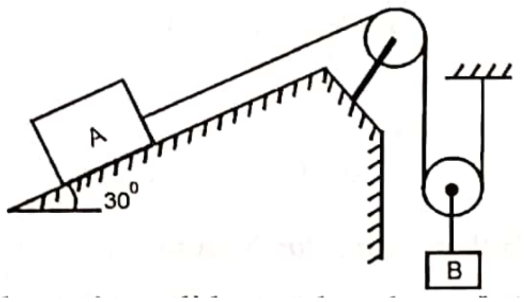
Using $\sum F_x = 0$ (Along the plane):

$$49.05 \sin 40^\circ - 0.2 \times (37.57) - 5a = 0$$

- $49.05 \sin 40^\circ$: Component of weight pulling the block down.
- $0.2 \times (37.57)$: Frictional force opposing motion (μN).
- $5a$: Inertia force acting opposite to the direction of acceleration.

or $a = 4.802\,\text{m/s}^2$ ...... **Ans.**

Q. A package A of mass 25 kg is being pulled up the incline by a load B of mass 60 kg connected to it by an inextensible rope passing over frictionless pulleys. Determine the accelerations of the two blocks and the tension in the connecting rope. Take $\mu_s = 0.4$ and $\mu_k = 0.3$ between the incline and A.



Solution:
Downward movement of load B causes package A to slide up the plane. Let us develop the relation between the accelerations of A and B using Constant String Length Method (CSLM).

Let variables x_A and x_B define the positions of A and B. As x_B increases, x_A would decrease. If L is the length of the string, then the length L is the sum of string portions in terms of x_A and x_B and plus/minus constants (constants are the string portions which don't change during motion like the length of cord wrapped over the pulleys).

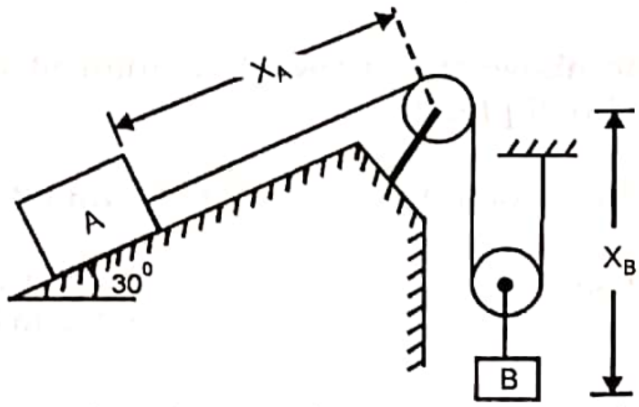
$$L = (2x_B) + (-x_A) \pm \text{constants}$$

Differentiating w.r.t time

$$0 = 2v_B - v_A$$

Differentiating again w.r.t time

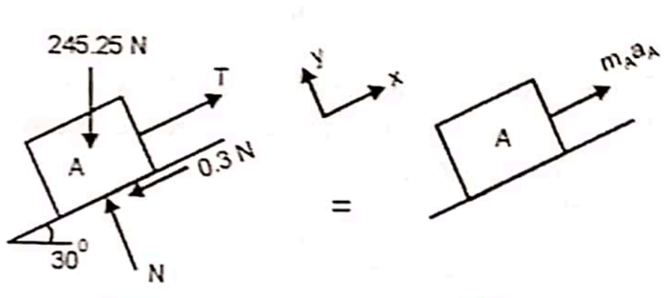
$$a_A = 2a_B \quad \ldots\ldots(1)$$



Let us isolate A and B and perform kinetic analysis of each of them.

Kinetics of Particles: Newton's Second Law

Kinetics of package A Applying equations of Newton's second law to A:



F.B.D. K.D.

Sum of forces in Y direction ($\Sigma F_y = m_A a_y$):

$$N - 245.25 \cos 30^\circ = 0$$

$$N = 212.39 \text{ Newton}$$

Sum of forces in Y direction ($\Sigma F_y = m_A a_y$):

$$N - 245.25 \cos 30^\circ = 0$$

$$N = 212.39 \text{ Newton}$$

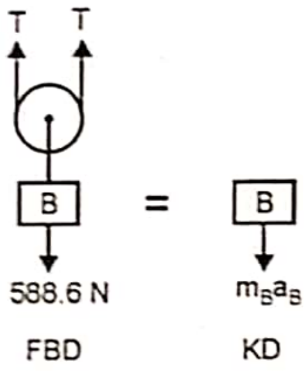
Sum of forces in X direction ($\Sigma F_x = m_A a_x$):

$$T - 245.25 \sin 30^\circ - 0.3N = m_A a_A$$

$$T - 245.25 \sin 30^\circ - 0.3(212.39) = 25 a_A$$

$$T - 186.34 = 25 a_A \quad \ldots\ldots (2)$$

Kinetics of load B Applying equations of Newton's Second Law to B:



Sum of forces in Y direction ($\Sigma F_y = m_B a_y$): *(Taking upward direction as positive)*

$$2T - 588.6 = -m_B a_B$$

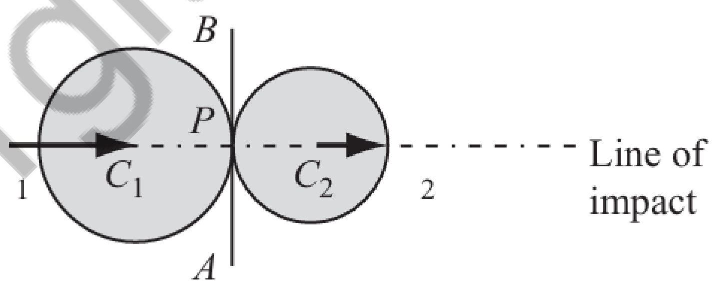
$$2T - 588.6 = -60a_B \quad \ldots\ldots(3)$$

Solving equations (1), (2) and (3), we get:

$a_A = 2.7\ \mathrm{m/s^2}$ **(Ans.)** $a_B = 1.35\ \mathrm{m/s^2}$ **(Ans.)** $T = 253.84\ \mathrm{N}$ **(Ans.)**

3. Impact Definitions

Q. Explain Direct Central Impact and Oblique Central Impact with diagram.
Solution:
Direct Central Impact:

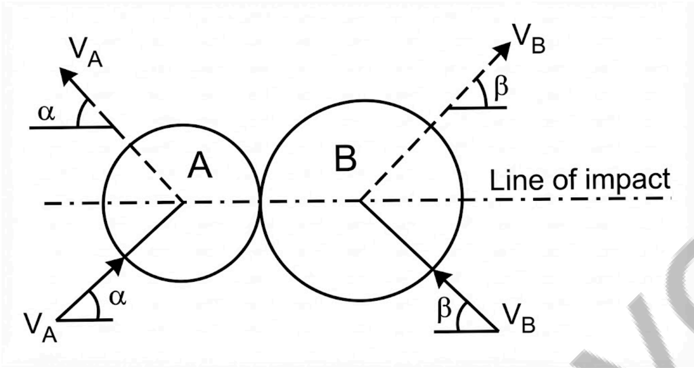


Direct Central Impact is a type of collision in which two bodies move along the same straight line (line of centres) before and after impact. The velocities of both bodies are collinear with the line joining their centres at the instant of collision, and the impulsive force during impact acts along this line only.

Key points:

- Motion is along a single straight line.
- Impact force acts along the line of centres.
- The bodies undergo deformation and restitution and then separate with new velocities.

Oblique Central Impact:



Oblique Central Impact is a type of collision in which the line of impact passes through the centres of the colliding bodies, but the initial velocity of one or both bodies is not along the line of impact. Hence, after collision, both the magnitude and direction of velocities change.

Key points:

- The impulsive force acts only along the line of impact.
- Velocity components along the line of impact change.
- Velocity components perpendicular to the line of impact remain unchanged.

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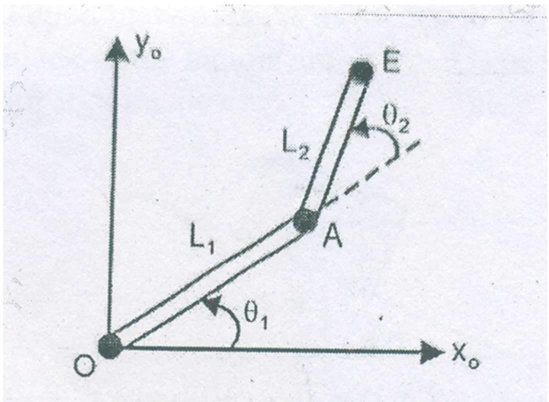
Module 6: Introduction to Robot Kinematics

1. Robotics Theory/Calculation

Q. A planar elbow manipulator is shown. Write the Denavit–Hartenberg (D–H) parameters and hence locate the position and orientation of the end point E of the manipulator. Given $L_1 = L_2 = 250 \text{ mm}, \quad \theta_1 = \theta_2$



Solution:

Given Data:

- Mechanism**:** Planar Elbow Manipulator (2-DOF)
- Link Lengths: $L_1 = 250 \text{ mm}$, $L_2 = 250 \text{ mm}$
- Joint Angles: $\theta_1 = 30^\circ$, $\theta_2 = 30^\circ$

D-H Parameters Table:
To establish the D-H parameters, we assign coordinate frames:

Frame 0: Attached to the base at O. z_0 is the axis of rotation perpendicular to the page.

Frame 1: Attached to joint A. x_1 aligns with link L_1.

Frame 2: Attached to the end-effector E. x_2 aligns with link L_2.

Since the manipulator is planar (2D):

Link Twist (α_i): 0° (all z-axes are parallel).

Link Offset (d_i): 0 (movement is purely in the x-y plane).

Link Length (a_i): Represents the length of the links (L_1 and L_2).

The D-H Parameter Table is:

Link (i)	Joint Angle (θi)	Link Offset (di)	Link Length (ai)	Link Twist (αi)
1	θ_1	0	L_1	0
2	θ_2	0	L_2	0

Transformation Matrices:

The general homogeneous transformation matrix for a link i is:

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix for Link 1 ($^0T_1$): Substituting $a_1 = L_1, \alpha_1 = 0, d_1 = 0$:

$$^0T_1 = \begin{bmatrix} \cos\theta_1 & -\sin\theta_1 & 0 & L_1 \cos\theta_1 \\ \sin\theta_1 & \cos\theta_1 & 0 & L_1 \sin\theta_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Transformation Matrix for Link 2 ($^1T_2$): Substituting $a_2 = L_2, \alpha_2 = 0, d_2 = 0$:

$$^1T_2 = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & L_2 \cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & L_2 \sin\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Transformation Matrix ($^0T_2$)

To find the position and orientation of point E with respect to the base O, we multiply the matrices:

$$^0T_2 = {}^0T_1 \times {}^1T_2$$

Using trigonometric identities ($\cos(\theta_1 + \theta_2) = c_{12}$ and $\sin(\theta_1 + \theta_2) = s_{12}$):

$$^0T_2 = \begin{bmatrix} \cos(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) & 0 & L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) & 0 & L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The **Position** vector is the last column:

$$P_x = L_1\cos\theta_1 + L_2\cos(\theta_1+\theta_2)$$
$$P_y = L_1\sin\theta_1 + L_2\sin(\theta_1+\theta_2)$$

The **Orientation** is determined by the sum of the angles $\theta_1 + \theta_2$.

Calculation of Position and Orientation:
Now, substitute the given values:

$L_1 = 250, L_2 = 250$

$\theta_1 = 30^\circ, \theta_2 = 30^\circ$

$\theta_1 + \theta_2 = 60^\circ$

Step A: Calculate Position (P_x, P_y)

$$P_x = 250 \cdot \cos(30^\circ) + 250 \cdot \cos(60^\circ)$$

$$P_x = 250(0.866) + 250(0.5)$$

$$P_x = 216.5 + 125$$

$$P_x = 341.5 \text{ mm}$$

$$P_y = 250 \cdot \sin(30^\circ) + 250 \cdot \sin(60^\circ)$$

$$P_y = 250(0.5) + 250(0.866)$$

$$P_y = 125 + 216.5$$

$$P_y = 341.5 \text{ mm}$$

Step B: Calculate Orientation: The orientation of the end-effector relative to the base X-axis is the sum of the joint angles:

$$\phi = \theta_1 + \theta_2$$

$$= 30^\circ + 30^\circ$$

$$= 60^\circ$$

Q. Classify Robot Mechanics and explain main parts of a robotic arm with neat sketch.
Solution:

Classification of Robot Mechanics:
Robots can be classified based on their mechanical structure and motion as follows:

1. Cartesian Robots (Rectangular Robots):
   - Movement along X, Y, and Z axes in straight lines.
   - Mainly used for pick and place, CNC machines.
2. Cylindrical Robots:
   - Movement in a cylindrical coordinate system (rotational about the base and linear along the arm).
   - Used for assembly operations.
3. Spherical (Polar) Robots:
   - Rotational movement about two axes and linear movement along one axis.
   - Used in handling at machine tools.
4. Articulated Robots:
   - Movements similar to a human arm with rotary joints.
   - Very flexible, widely used in welding, painting.
5. SCARA Robots:
   - Selective Compliance Assembly Robot Arm.
   - Combines rotational and linear movements in a horizontal plane.
   - Used in assembly operations.

Main Parts of a Robotic Arm:

- Base: Supports the robot and allows rotation.
- Actuators: Motors that drive the robot joints.
- Joints: Provide rotational or translational movement, usually rotary.
- Links: Rigid segments between joints.
- End Effector: Tool attached to the last link, interacts with the environment (grippers, welding torches).
- Sensors: To gather information about position, force, and environment.