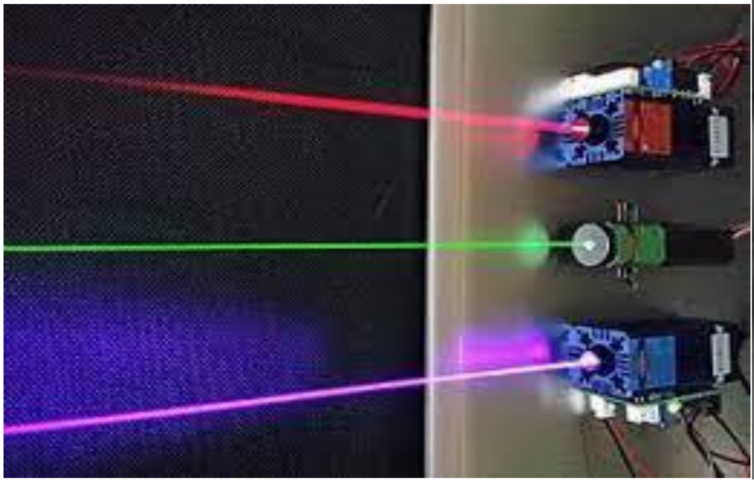
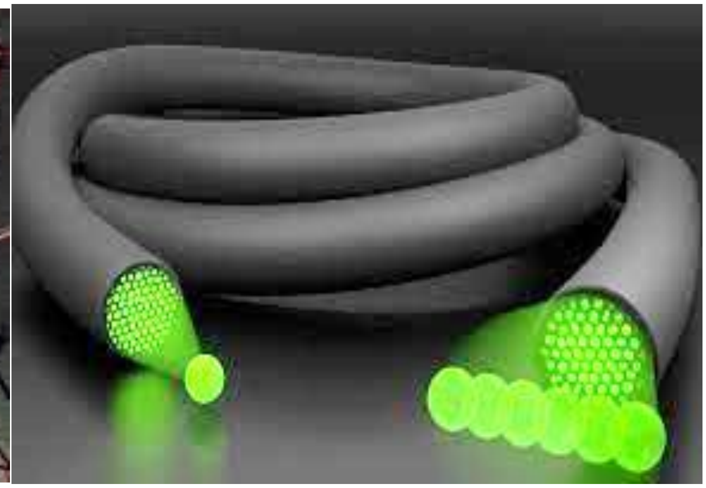


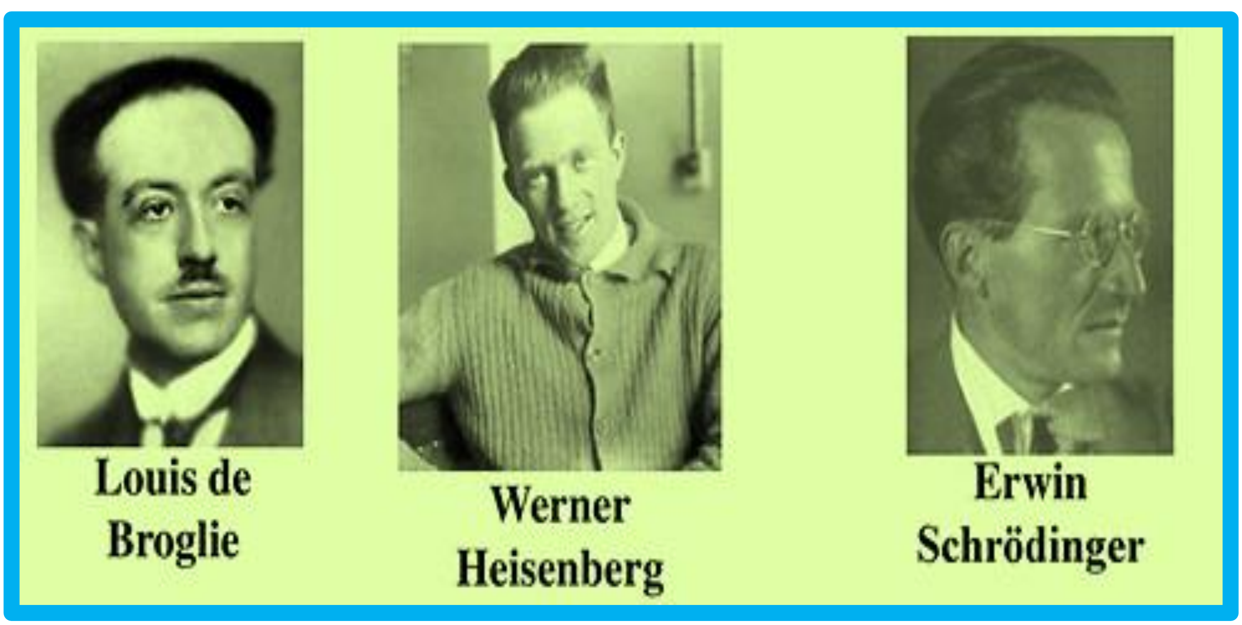


FE SEM-I 2024-25

COURSE NAME: APPLIED PHYSICS

COURSE CODE: BSC102

(As per NEP-2020 revised syllabus)



Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned			
		Theory	Pract.	Tut.	Theory	Pract.	Tut.	Total
BSC102	Applied Physics	2	--	-	2	--	-	2

Course Code	Course Name	Theory					Term work	Pract / Oral	Total
		Internal Assessment Test (IAT)			End Sem Exam	Exam Duration (in Hrs)			
		IAT-I	IAT-II	IAT-I + IAT-II (Total)					
BSC102	Applied Physics	15	15	30	45	02	--	--	75

Rationale:

Most of the engineering branches are being off-spring of basic sciences where physics is playing a pivotal role in concept and understanding of foundation of core engineering branches. This syllabus is developed by keeping in mind, needs of all branches that we offer in University of Mumbai. In the distribution of modules, core physics and its applied form are given priority. Further, it is ensured that these modules will cover prerequisites needed for engineering courses to be introduced in higher semesters as core subjects or as interdisciplinary subjects in respective branches.

Course Objectives:

1. To provide students with a basic understanding of laser operation.
2. To explain the basic working principle of Optical fiber and its use in communication technology.
3. To demonstrate principles of interference in thin film.
4. To describe Maxwell's equations and their significance.
5. To build a foundation of quantum mechanics needed for modern technology.
6. To give exposure to the concept of Fermi level in semiconductors.

Course Outcomes:

1. Learners will be able to ILLUSTRATE the use of laser in LIDAR and Barcode reading.
2. Learners will be able to APPLY the foundation of fiber optics in the development of modern communication technology
3. Learners will be able to determine the wavelength of light and refractive index of liquid using the interference phenomenon.
4. Learners will be able to ARTICULATE the significance of Maxwell's equations.
5. Learners will be able to RELATE the foundations of quantum mechanics with the development of modern technology.
6. Learner will be able to CLASSIFY semiconductors and EXPLAIN variation of Fermi level with temperature and doping concentration.

DETAILED SYLLABUS:

	Name of Module	Detailed Content	Hours	CO Mapping
	Prerequisite	Basic knowledge of optics and atomic structure, Wave front and Huygens principle, reflection and refraction, Interference by division of wave front, Refractive index of a material, Snell's law, Basics of vector algebra, partial differentiation concepts, Dual nature of radiation, Photoelectric effect, Matter waves, Davisson-Germer experiment. Intrinsic and extrinsic semiconductors, electrical resistivity and conductivity concepts	--	--
I	**Lasers**	Lasers: Spontaneous and stimulated emission, population inversion, pumping, active medium & active center, resonant cavity, coherence length and coherence time, Characteristics of lasers, He-Ne laser: construction and working. Fiber laser Construction and working Application : (i)Elementary knowledge of LIDAR(ii) Barcode reader (iii) Application of laser in metal work	**04**	**CO1**
II	**Fibre Optics**	Optical fibers: Critical angle, acceptance angle, acceptance cone, numerical aperture, total internal reflection and propagation of light, Types of optical fibers: Single mode & multimode, step index & graded index, attenuation, attenuation coefficient, factors affecting attenuation, Fibre Optic Communication System, Advantages of optical fiber	**04**	**CO2**

		communication, numerical		
III	Interference In Thin Films	Interference in thin film of uniform thickness, conditions of maxima and minima for reflected system, Conditions for maxima and minima for wedge shaped film (qualitative), engineering applications – (i) Newton’s rings for determination of unknown monochromatic wavelength and refractive Index of transparent liquid (ii) Anti Reflecting Coating	**04**	**CO3**
IV	**Electrodynamics**	Vector Calculus: Gradient, Divergence, Curl. Gauss’s law, Amperes’ circuital Law, Faraday’s law, Divergence theorem , Stokes theorem Maxwell’s equations in point form, Integral form and their significance(Cartesian coordinate only)	**04**	**CO4**
V	**Quantum Physics**	de Broglie hypothesis of matter waves, de Broglie wavelength for electron, Properties of matter waves, Wave function and probability density, mathematical conditions for wave function, problems on de Broglie wavelength, Need and significance of Schrödinger’s equations, Schrödinger’s time independent and time dependent equations, Energy of a particle enclosed in a rigid box and related numerical problems, Quantum mechanical tunneling, Principles of quantum computing: concept of Qubit.	**06**	**CO5**
VI	Basics Of Semiconductor Physics	Direct and Indirect Band Gap Semiconductors, Electrical Conductivity of Semiconductors, Drift Velocity, Mobility and Conductivity in Conductors Fermi- Dirac distribution function, Position of Fermi Level in Intrinsic and Extrinsic Semiconductors.	**04**	**CO6**

Text Books:

1. A Text book of Engineering Physics -Dr. M. N. Avadhanulu, Dr. P. G. Kshirsagar, S. Chand, Revised Edition 2014
2. Modern Engineering Physics - A. S. Vasudeva, S. Chand, Revised Edition 2013
3. Engineering Physics D. K Bhattacharya, Poonam Tandon, Oxford Higher Education, 1st Edition 2015
4. Engineering Physics -R. K. Gaur,S. L. Gupta, DhanpatRai Publications, 2012
5. Engineering Physics -V. Rajendran, McGraw Hill Educations, 2017
6. A Textbook of Nanoscience and Nanotechnology, T. Pradeep Tata McGraw Hill Education Pvt. Ltd., 2012

References:

1. Concepts of Modern Physics - ArtherBeiser, ShobhitMahajan, S. Choudhury, McGraw Hill, 7thEdition 2017
2. Fundamentals of optics - Francis A. Jenkins, Harvey E. White, McGraw Hill Publication, India, 4th Edition
3. Fundamentals of Physics, Halliday and Resnick, Wiley publication
4. Introduction to Electrodynamics, D. J. Griffiths, Pearson PublicationOnline

References:

Sr. No.	**Website Name**
1.	https://archive.nptel.ac.in/courses/115/102/115102124/
2.	https://archive.nptel.ac.in/courses/115/102/115102025/
3.	https://archive.nptel.ac.in/courses/115/105/115105132/

Assessment:

Internal Assessment Test (IAT) for 15 marks each:

- IA will consist of Two Compulsory Internal Assessment Tests. Approximately 40% to 50% of the syllabus content must be covered in the IAT-I and the remaining 40% to 50% of the syllabus content must be covered in the IAT-II.

End Semester Theory Examination:

Question paper format

- Question Paper will comprise a total of **five questions each carrying 15 marks Q.1** will be **compulsory** and should **cover the maximum contents of the syllabus**

- **Remaining questions** will be **mixed in nature** (part (a) and part (b) of each question must be from different modules. For example, if Q.2 has part (a) from Module 3 then part (b) must be from any other Module randomly selected from all the modules) A total of **three questions** need to be answered

Course Code	Course Name	Teaching Scheme (Contact Hours)			Credits Assigned			
		Theory	Pract.	Tut.	Theory	Pract.	Tut.	Total
BSL101	**Applied Physics Lab**	--	1	-	--	0.5	-	0.5

Course Code	Course Name	Theory					Term work	Pract / Oral	Total
		Internal Assessment Test (IAT)			End Sem Exam	Exam Duration (in Hrs)			
		IAT-I	IAT-II	IAT-I + IAT-II (Total)					
BSL101	**Applied Physics Lab**	--	--	--	--	--	25	--	25

Lab Objectives:

1. To develop scientific understanding of the physics concepts.
2. To develop the ability to explain the processes and applications related to science subjects.
3. To apply skills and knowledge in real life situations.
4. To improve the knowledge about the theory concepts of Physics learned in the class.
5. To improve ability to analyze experimental result and write laboratory report.
6. To develop understanding about inferring and predicting.

Lab Outcomes: Learners will be able to..

1. Determine wavelength / divergence of laser beam.
2. Determine parameters like numerical aperture / power attenuation of an optical fibre.
3. Perform experiments based on interference in thin film and determine radius of curvature of lens / diameter of wire / thickness of paper.
4. Calculate basic parameters / constants using semiconductors.
5. Determine energy gap / resistivity of a semiconductor.
6. Learner to understand the concept for virtual lab as per syllabus.

List of Experiments. (Minimum five experiments required)

Sr No	List of Experiments	Hrs	LO Mapping
01	Determination of wavelength using Diffraction grating. (Laser source)	**01**	**LO1**
02	Study of divergence of laser beam	**01**	**LO1**
03	Determination of Numerical Aperture of an optical fibre.	**01**	**LO2**
04	Measuring optical power attenuation in your plastic optical fiber	**01**	**LO2**
05	Determination of radius of curvature of a lens using Newton's ring set up.	**01**	**LO3**
06	Determination of diameter of wire/hair or thickness of paper using Wedge shape film method.	**01**	**LO3**
07	Determination of 'h' using photo cell	**01**	**LO4**
08	Determination of 'h' using LED	**01**	**LO4**
09	Determination of energy band gap of semiconductor.	**01**	**LO5**
10	Determination of resistivity by four probe method.	**01**	**LO5**
11	Any other experiment based on syllabus may be included, which would help the learner to understand concept. Virtual lab may be developed and used for performing the experiments , after defining a suitable LO	**01**	**LO6**

Term Work Marks: 25 Marks (Total marks) = 10 Marks (Experiment) + 10 Marks Project + 5 Marks (Attendance)

Project work will be extended to semester-2 as well. In semester 1, a group of four students will be formed; a domain may be provided by faculty, the group will frame a problem statement in consultation with faculty. A PPT presentation with problem statement, preliminary literature survey, execution plan and a probable outcome is to be considered for awarding marks. Proper rubrics must be framed by faculty member

LASER

(Light Amplification by Stimulated Emission of Radiation)

Lasers: Spontaneous and stimulated emission, population inversion, pumping, active medium & active center, resonant cavity, coherence length and coherence time, Characteristics of lasers, He-Ne laser: construction and working. Fiber
laser Construction and working Application : (i)Elementary knowledge of LIDAR(ii) Barcode reader (iii) Application of laser in metal work

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Q. What is laser? How laser light is differing from ordinary light?

Laser means **L**ight **A**mplification by **S**timulated **E**mission of **R**adiation. Laser is a source of light which emits highly intense, monochromatic, coherent, collimated and directional beam of radiation.

The laser light is differed from ordinary light according to

1. **Intensity: -** The laser beam travel as a narrow beam. Therefore, large power is concentrated in a narrow beam. Hence intensity of laser beam is high as compare to ordinary light beam.
2. **Monochromatic: -** The laser light consists of almost single value of wavelength. All the photons are of same wavelength. Hence laser beam is highly monochromatic.
3. **Coherent:** - The laser light waves are in phase with each other. Hence laser light is highly coherent. The coherent length of laser beam is of the order of few kilometers.
4. **Collimated: -** The laser light waves are parallel to each other and highly focused. The laser beam will not spread even if it travels very large distance.
5. **Directional: -** The laser beam is emitted only in one direction along the axis of the source. The laser radiation is intense, coherent and collimated. Therefore, laser beam can cover large distance with negligible diffraction. Therefore, directionality of laser radiation is very high as compared to ordinary light radiation.

Q. Comments on Intensity & Directionality of laser beam?

- The laser light is always producing in the form of radiation.
- All the photon energy is concentrated in the form of narrow beam. Also, the laser light waves are highly monochromatic & coherent.
- The coherent length of laser beam is of the order of few kilometers.
- Therefore, laser beam can cover large distance with negligible diffraction. Hence laser light is highly directional & intense.

Q. Explain the three process of interaction between radiation and the particles to produce the laser beam?

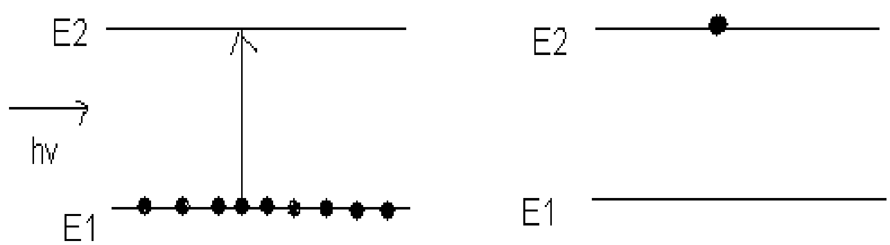
Q. Explain the following term in case of laser and define Einstein's Co-efficient?

a) Absorption of radiation b) Spontaneous emission c) Stimulated emission.

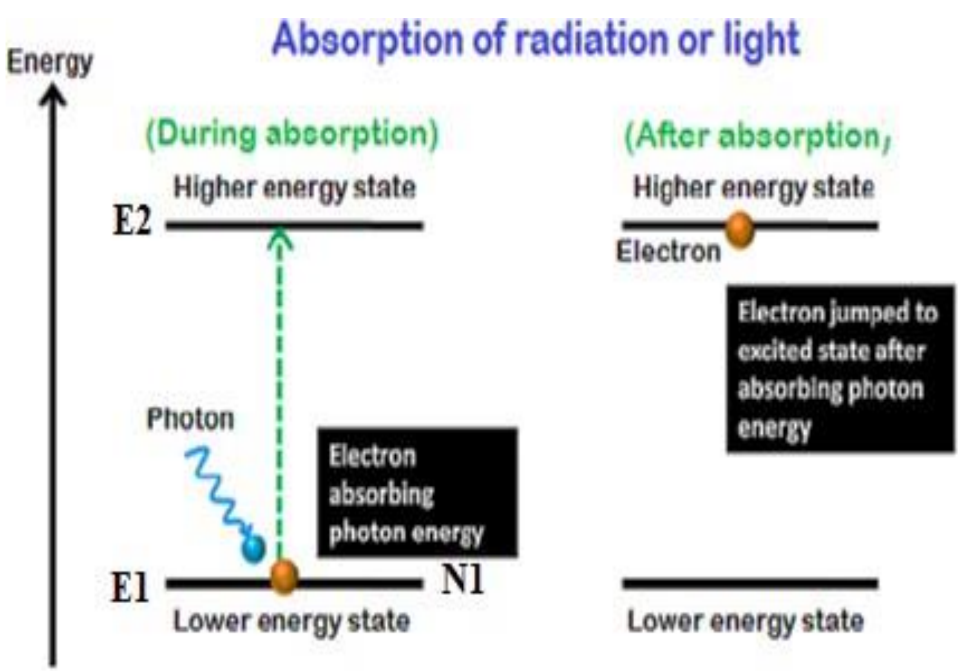
There are three process of interaction between the radiation (photons) and the particle (electrons, atoms, molecules etc.) to produce the laser beam.

- Absorption.
- Spontaneous emission.
- Stimulated emission.

1) Absorption of incident energy:-



Absorbtion of incident energy



The atoms or molecules of the substance are always in the ground state (lower energy state E1). When the energy (light, heat, electrical or chemical energy) is incident on the substance, the atoms absorb this energy and make transition in to the excited state (Higher energy state E2). This is known as absorption.

If N_1 – no. of atoms in the ground state. Q – Incident energy density.

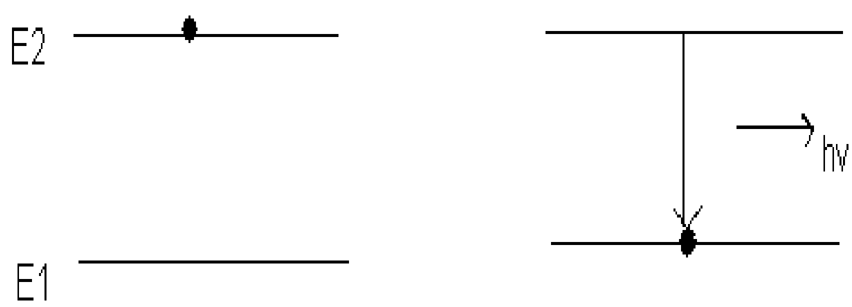
The probability of absorption is directly proportional to N_1 and Q.

$$P_{ab} \propto N_1 Q$$

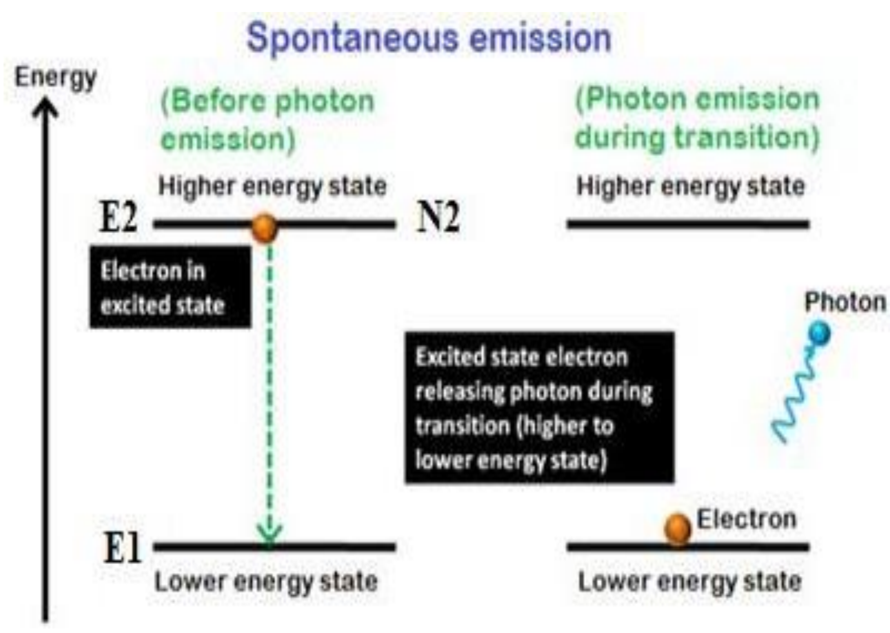
$$P_{ab} = B_{12} N_1 Q$$

Proportionality constant B_{12} is known as Einstein's coefficient of absorption.

2) Spontaneous emission:-



Spontaneous emission



The life time of the atoms to remain in the excited state is very short of the order of 10^{-8} second due to electrostatic force of attraction between positively charged nucleus and negatively charged electrons.

Therefore, the excited atoms again jump in to the ground state. During this transition a photon of energy

$$h\nu = E_2 - E_1$$ is emitted spontaneously.

The emitted photons are random in phase and direction. Therefore, the emitted light is highly incoherent.

If N_2 – no. of atoms in the excited state.

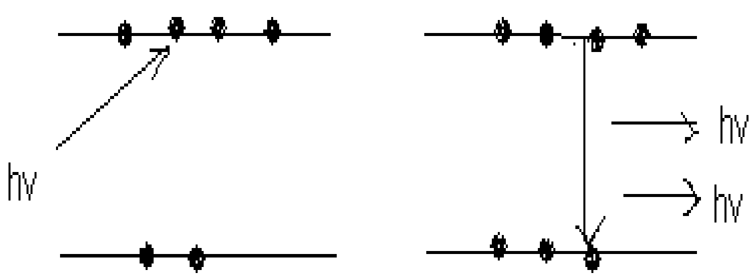
The probability of spontaneous emission is directly proportional to N_2.

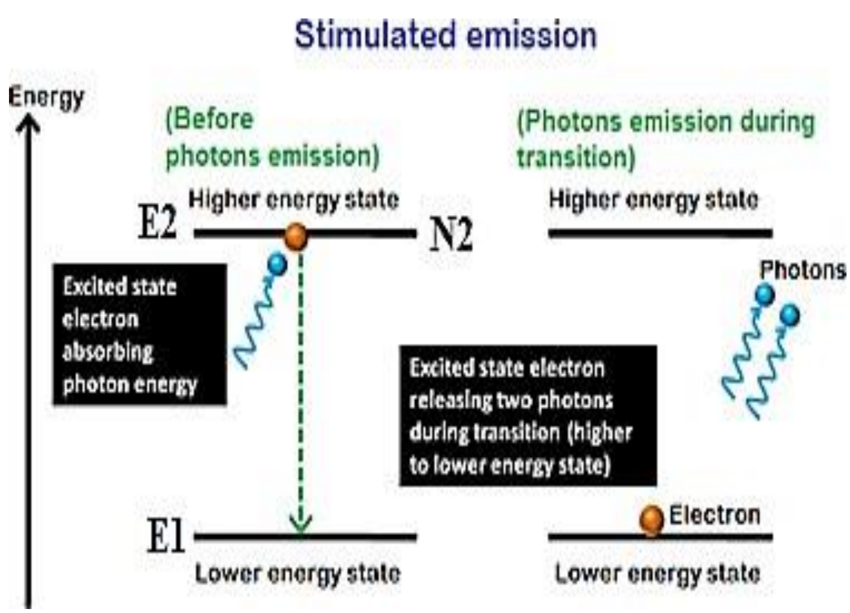
$$P_{sp} \propto N_2$$

$$P_{sp} = A_{21} N_2$$

Proportionality constant A_{21} is known as Einstein's coefficient of spontaneous emission.

3) Stimulated emission of radiation: -





If the numbers of atoms in the excited state are more than the number of atoms in the ground state, then spontaneously emitted photons take the collision with the excited atom and force this atom to jump in to the ground state. During this transition a photon which is identical to incident photon is emitted. The both photons are identical in all respect. Thus, spontaneously emitted photons stimulate the number of other photons which are identical in their direction, phase, wavelength, and amplitude known as light amplification by stimulated emission of radiation (LASER). **Emitted light is highly coherent**.

If N_2 – no. of atoms in the ground state.

Q – Incident energy density.

The probability of stimulated emission is directly proportional to N_2 and

Q. $P_{st} \alpha N_2 Q$

$$P_{st} = B_{21} N_2 Q$$

Proportionality constant B_{21} is known as Einstein's coefficient of stimulated emission.

According to law of conservation of

energy Upward transition = Downward

transition $P_{ab} = P_{sp} + P_{st}$

$$\mathbf{B_{12}N_1Q = A_{21}N_2 + B21\ N_2\ Q}$$

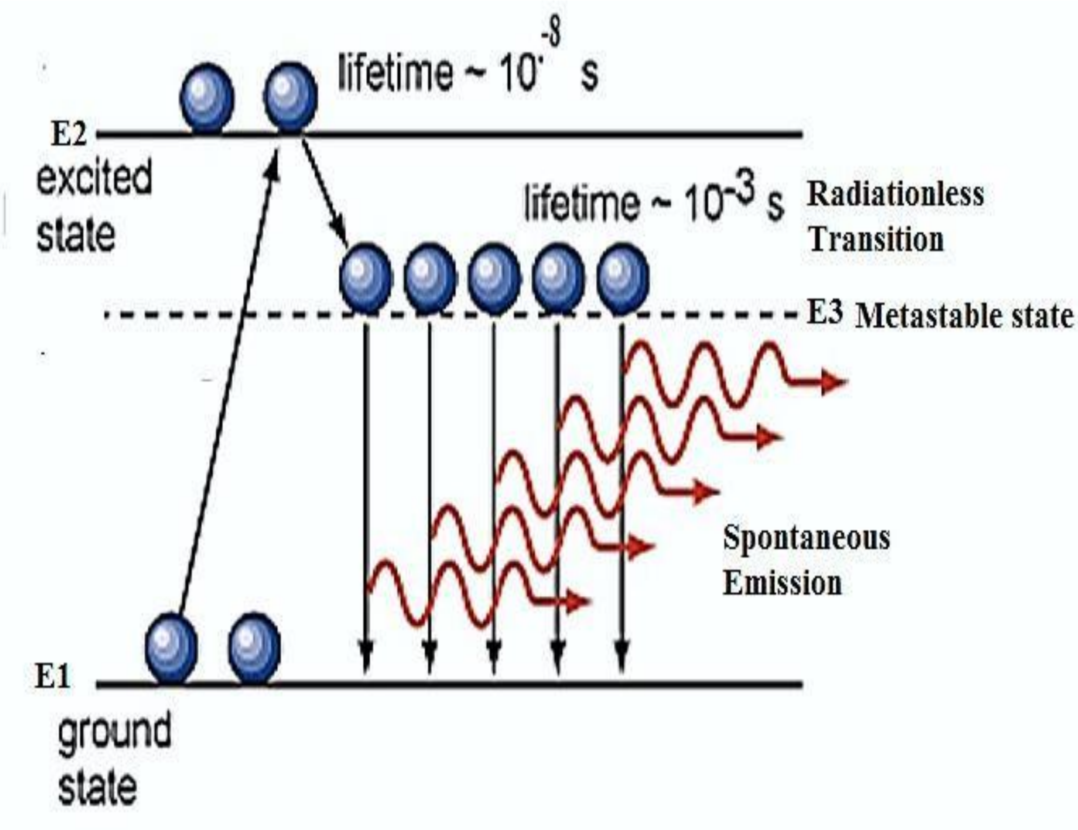
This is known as Einstein equation and B_{12}, B_{21}, B_{21} are known as Einstein coefficient.

Q.2. Distinguish between spontaneous & stimulated emission of radiation?

Spontaneous emission	Stimulated emission
1. It is natural process.	It is forced emission.
2. It is takes place within 10^{-8} sec.	It is takes place within 10^{-3} sec.
3. Emitted light is incoherent.	Emitted light is highly coherent.
4. It is not controllable	It can be controlled externally.
5. Used in ordinary source of light	Used in laser light source.
6. Population inversion of atoms is not required.	Population inversion of atoms is required.

Q.3. What is population inversion & Pumping? Explain different types of pumping methods?

Population Inversion:-



Amplification of light requires that number of atoms in excited state must be greater than number of atoms in the ground state. The situation in which the higher energy state of the material is more populated than the lower energy state is called population inversion.

- The material used for laser source may be solid, liquid or gas. The region in which the state of population inversion is achieved is called active medium. The actual laser light is emitted from this region.
- The atoms, ions or molecules of the material that takes part to produce the laser light are called active centers.
- For population inversion, atoms must have stay in the excited state for longer time. But excited atoms have natural tendency to rapidly de-excite to their ground state within 10^{-8} second.
- Therefore, population inversion is not possible in the material having only two energy state

  i.e ground state and excited state.
- The population inversion is achieved in the material in which higher energy state just below the excited state known as metastable state exist.
- The life time of the excited atoms to stay in the metastable state is of the order of 10^{-3} second. Thus, inversion of population of atoms is possible in metastable state.

Pumping:

The energy has to be supplied to the material from the outside, so that the atoms can absorb this energy and continuously raised to the excited state to achieve the population inversion. The process of supplying energy to the material or system to achieve the state of population inversion is called pumping. There are two types of pumping process.

- Continuous pumping process: - The pumping energy is continuously supplied to the material till the laser source is ON. The continuous laser beam is produced. The cooling arrangement is required to remove the excess heat generated in the system.
- Pulsed pumping process: - The pumping energy is supplied at regular interval of time to the material. The pulsed laser beam is produced.

Pumping Methods:

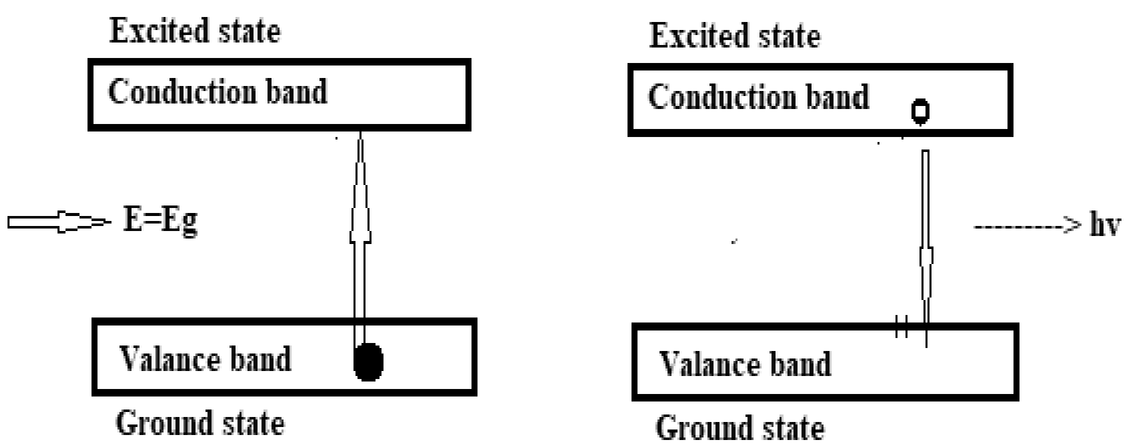
- Optical pumping: - A source of light (photons) is used to supply excitation energy. E.g. Xenon flash lamp, discharge tube etc.
- Electrical pumping: - The electric field is used to supply excitation energy. The collisions of electrons with atoms excite the atoms in to the higher energy state.
- Heat pumping: - The excitation energy is provided in the form of heat. The material is heated to sufficient temp. to achieve the population inversion.
- Chemical pumping: - The exothermic chemical energy is used to provide the excitation energy.

Q.4. What is metastable state? Explain different types of pumping schemes? Why four level pumping scheme is efficient?

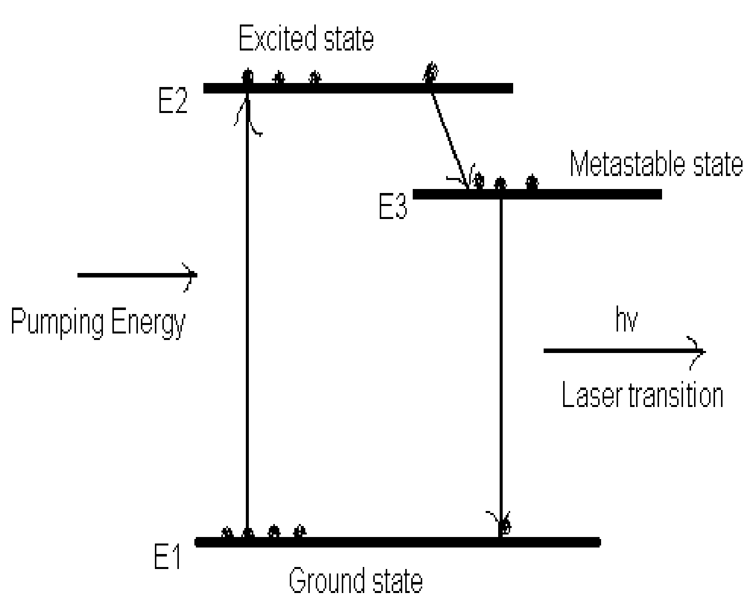
The higher energy state just below the excited state in which atoms can stay up to certain millisecond and population inversion condition is achieved is known as metastable state.

Pumping Scheme: -

- **Two level pumping schemes: -**

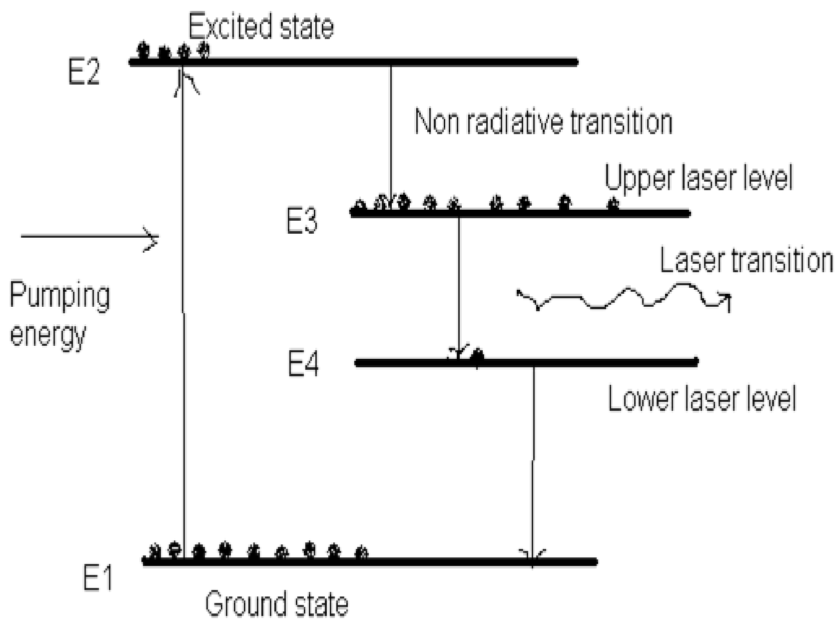


- The two level pumping scheme is exist only in direct band gap semiconductor materials like Ga, As etc. The lower energy state is valence band and higher energy state is conduction band. The electrons are transferred from valence band to conduction and they come back to valence band giving laser transition.
- **Three level pumping schemes: -**



The material or system in which three energy level are involved is called three level pumping schemes. The atoms absorb the excitation energy and raised to excited state E2. The non-radiative transition of atoms takes place from excited state E2 to metastable state E3 and population inversion takes place in metastable sate. The laser transition takes place from metastable state E3 to ground state E1. The energy levels in which laser transition takes place are called upper lasing level and lower lasing level.

- **Four level pumping schemes: -**

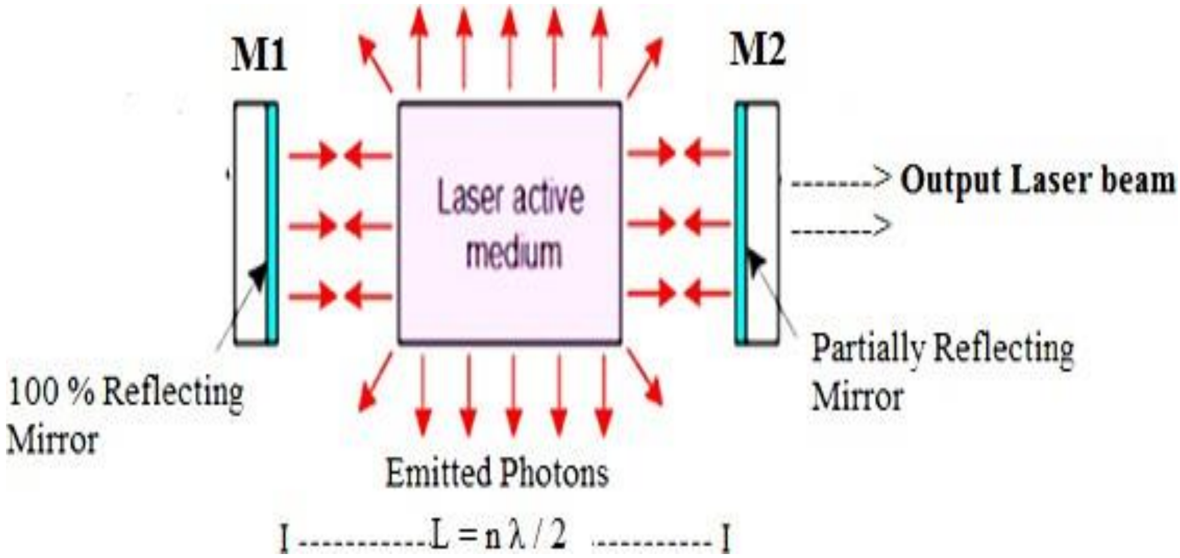


Four energy levels are involved in this pumping scheme. The atoms absorb the excitation energy and jumped from ground state E1 to excited state E2. The non-radiative transition of atoms takes place from excited state E2 to metastable state E3 and population inversion takes place in metastable sate. The laser transition takes place from metastable state E3 to lower energy state E4. The balance of excitation energy is lost due to collision and atoms are de-excited to ground state E1.

The population inversion is achieved between two metastable states; therefore, emitted photon has definite energy & wavelength, hence four level pumping scheme is more efficient than three level pumping schemes.

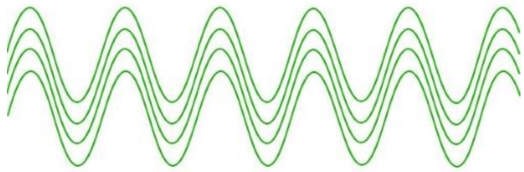
Q. What is use of optical resonator in laser? Explain plane parallel optical resonator?

Optical Resonator or Resonance Cavity:



- The numbers of photons that are identical in all respect are produced due to stimulated emission in the active medium. But these photons are scattered in all directions like light emitted by ordinary source of light.
- The laser light requires light waves (photons) having same wavelength, same phase, same polarization and emitted in same direction. This number of identical light waves should increase progressively.
- The resonance cavity is formed using two reflecting surface or mirror. The active medium in which lasing action takes place is kept between two reflecting mirrors M1 and M2. The mirror M1 is fully reflecting and other mirror M2 is partially reflecting.
- The light waves moving along the axis of laser source are retained due to reflection and other light waves that are scattered in various directions are lost. The photons that are bouncing back and forth between two reflecting mirrors stimulate the no. of other photons and no. of identical wave's increases.
- The to and fro distance covered by the light waves between mirrors M1 and M2 must be integral multiple of wavelength of the photons to form the optical standing wave (stationary wave) due to constructive interference. $2L = n\lambda$, therefore resonating length $L = n\lambda / 2$. The laser beam comes out from partially reflecting mirror M2.

Coherence length and coherence time



In laser radiation, all the photon waves are in phase to each other, therefore laser radiation is highly coherent.

Coherence Length:

Coherence length is the distance that light can travel while maintaining its phase relationship. It is defined as the distance travelled by the laser light over which the coherence of laser radiation significantly decays. This phenomenon occurs when two or more photon waves are traveling in the same direction, and their peaks and valleys align.

In other words, coherence length is a measure of how far a beam of light can travel while remaining coherent. It is a property of the light source and depends on the size and nature of the emitting object.

In practical terms, the resolution of an optical imaging system is determined by coherence length. If the coherence length is shorter than the distance that the light needs to travel through a sample, then the resulting image will be blurry and distorted. By contrast, longer coherence lengths can provide clearer images and higher-resolution measurements.

The coherence length (L_c) is related to the wavelength (λ) and the degree of spectral bandwidth (Δλ) of the wave by the formula:

$$L_c = \frac{\lambda^2}{\Delta\lambda}$$

A smaller spectral bandwidth or a longer wavelength results in a longer coherence length. Coherence length is important in various applications, including interferometry, holography, and optical communication systems.

Coherence Time:

The time for which coherence length of laser radiation remains same is known as **coherence time**. Coherence time is a measure of the time duration over which a wave maintains its coherence or phase relationship.

For example, in the case of light-emitting diodes, the coherence time is half a picosecond, while the associated coherence length is approximately 15 microns. Conversely, in case of basic laser the coherence time is half a nanosecond, while the associated coherence length of around 15 centimeters.

The coherence length & coherence time are inversely proportional to spectral band width.

$$\mathbf{Length_{coherence}} = \mathbf{c \; x \; Time_{coherence}} = \frac{\lambda^2}{\Delta\lambda}$$

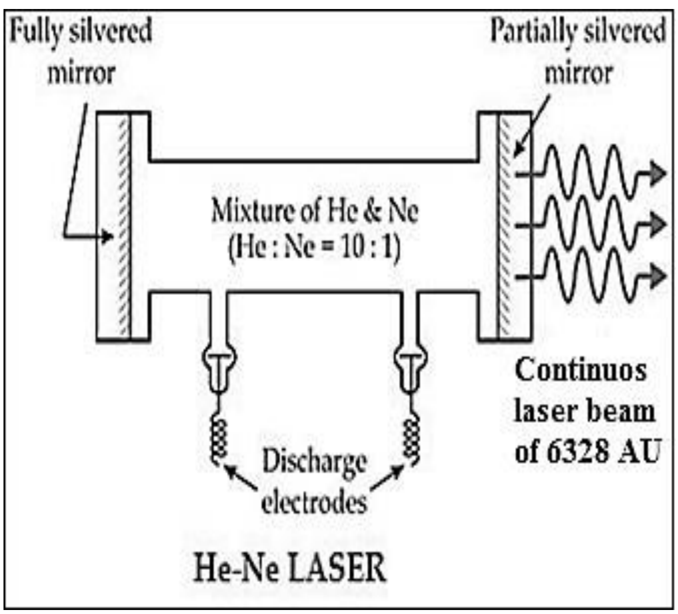
The coherence time (T_c) can be related to the spectral line width (Δv) of the wave by the formula:

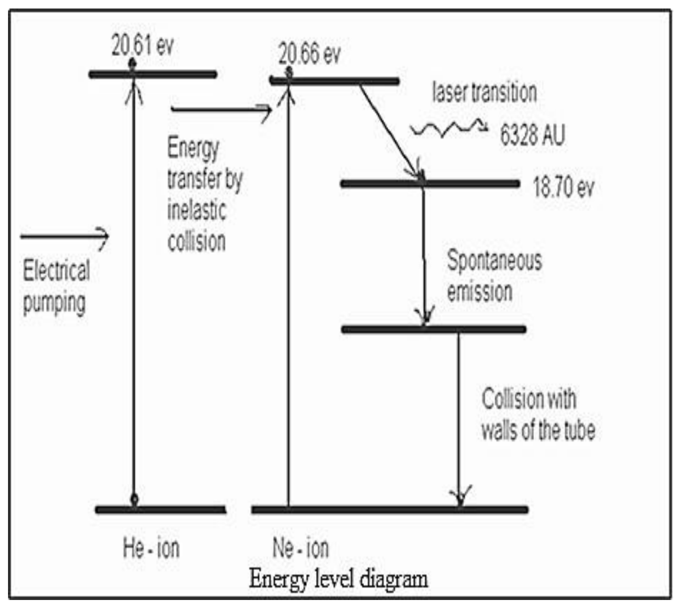
$$\mathbf{Time_{coherence}} = \frac{1}{\Delta\lambda}$$

Similar to coherence length, a smaller spectral line width leads to a longer coherence time. Coherence time is a important parameter in fields such as quantum optics, nuclear magnetic resonance (NMR) spectroscopy, and quantum cryptography.

Q. Explain the construction and working of He-Ne laser with energy level diagram? Comment on its efficiency?

He – Ne laser:





CONSTRUCTION:

- He-Ne laser is a first gas laser prepared by scientist hall and his coworkers in 1961.
- It consists of long narrow discharge tube of length 10 cm to 80 cm and diameter 0.5 cm.
- The discharge tube contains mixture of He and Ne gas in the ratio of 10:1 with a pressure of 0.1 and 1 mm of Hg respectively. The 'He' gas is an active medium and Ne ions are the active centers.
- The two electrodes are fitted to discharge tube which is connected to high voltage power

supply. Thus, electrical pumping is used to provide the excitation energy.

- The discharge tube is placed between two mirrors M1 and M2 which forms resonance cavity. The mirror M1 has fully reflecting surface while mirror M2 has partially reflecting surface.

WORKING: -

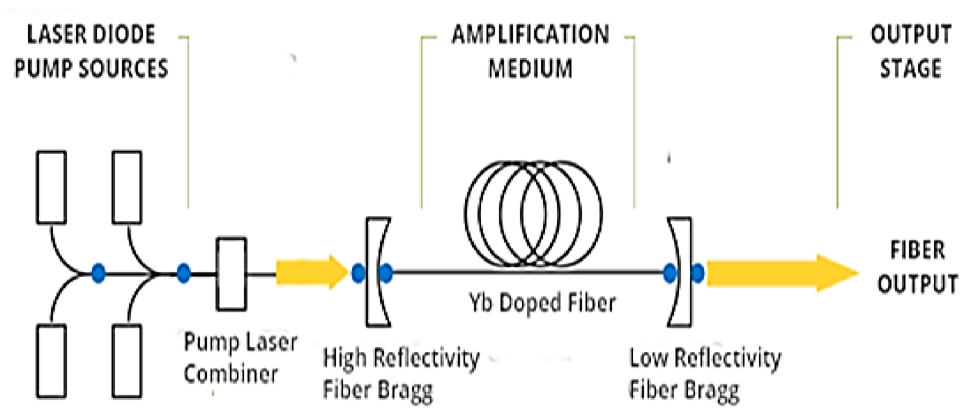
- When power supply is ON, the electric discharge is produced and the electrons produced in the discharge tube collides with the He and Ne ions. By absorbing the electrical energy, the He and Ne ions are excited to higher energy state.
- The metastable state of He ions is at 20.61 ev and for Ne ions at 20.66 ev above the ground state.
- Since the He ions are lighter than Ne ions, first He ions are excited to their metastable state at 20.61 ev. The density of He ions is more than Ne ions (for every Ne ion there are 10 He ions in the discharge tube) and He ions has good thermal conductivity. The excited He-ions collide with the

  Ne-ions and transfer their energy to Ne-ions by inelastic collision. By absorbing the energy difference of 0.05 ev from the He ions, the Ne ions are excited to their metastable state at 20.66 ev. **Thus, He ions help for the population inversion of Ne ions as well as He gas acts as coolant due to its good thermal conductivity.**
- When Ne ions de excite from metastable state at 20.66 ev to next lower energy state at 18.70 ev, then laser transition takes place and a photon of wavelength 6328 AU is emitted spontaneously. The laser is operated at wavelength 6328 AU by using resonant cavity.
- The balance of excitation energy is lost by spontaneous emission and collision with the walls of the discharge tube.
- The spontaneously emitted photons bouncing back and forth between two reflecting mirrors and stimulates the number of other photons. The number of light waves that are identical in phase, wavelength, amplitude and direction go on increasing. The distance between the two mirrors are adjusted to integral multiple of half the wavelength of emitted photons, so that optical standing wave is set up inside the discharge tube and continuous laser beam comes out through partially reflecting mirror M2.
- **Efficiency: -**The ratio of output optical power to the input power required to operate the laser is known as efficiency of laser. The efficiency of He-Ne laser is less (1 mW to 50 mW), but it is popular due to directionality and coherence. Four level pumping scheme is used. The emitted light is highly directional & coherent; therefore, this laser is used for research applications.

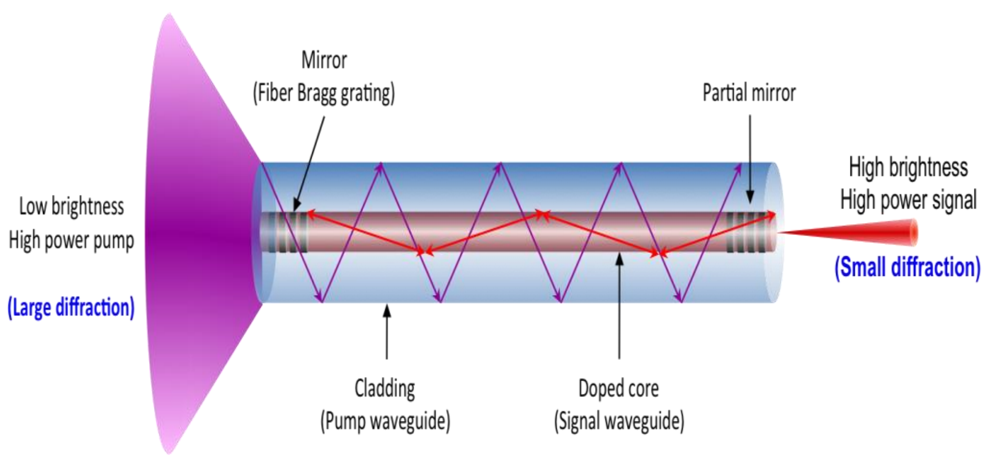
Summary:

- **Types of laser:** Gas laser
- **Active material:** Mixture of He-Ne gas.
- **Active centers:** Ne ions.
- **Pumping scheme:** Four level pumping scheme.
- **Pumping method:** Electrical pumping.
- **Pumping process:** Continuous pumping.
- **Laser output:** Continuous laser beam of wavelength 6328 AU.
- **Efficiency:** Low.
- **Output power:** 1 mW to 50 mW.
- **Laser output:** Emitted laser beam is highly directional & coherent.
- **Applications:** In research field, Barcode scanners, Tool alignment, Non-contact measuring and monitoring, Blood analysis, Particle counting and food sorting

Fiber laser:



Schematic diagram of fiber laser



Fiber lasers principles:

Fibre lasers are a type of solid-state lasers that use optical fibres as their active gain medium. In these lasers, a fibre made of silicate or phosphate glass absorbs raw light from the pump laser diodes and transforms it into a laser beam with a specific wavelength. To achieve this, the optical fibre is doped by different rare-earth doping elements and laser beams can be created with a wide range of wavelengths. Some common rare-earth doping elements and their emitted wavelengths are neodymium (780-1100nm), ytterbium (1000-1100nm), praseodymium (1300nm), erbium (1460-1640nm), thulium (1900-250nm), holmium (2025-2200nm), and dysprosium (2600-3400nm).

The well collimated high power laser light is produced by five stages

- Creation of pump light
- Collection and travel into the optical fibre
- Pump light passes through the optical fibre
- Stimulated emission in the laser cavity
- Amplification of raw laser light into a laser beam

Creation of pump light

In fibre lasers, electricity is used as the energy source. Multiple Pump laser diodes convert electrical energy into light energy of specific wavelengths.

Collection and travel into the optical fibre

A coupler combines the light from multiple laser diodes into one. This coupler is a part of the optical fibre. It has multiple entry points on one side, each of which connects to a fibre from an individual laser diode. On the other side, there's a single exit point that connects to the main fibre. Once all the light is collected, it travels to the laser medium.

Pump light passes through the optical fibre. In the next stage, the laser diodes light flows through the optical fibre to the laser medium. The fibre consists of two main components: the core and the cladding. The core is made of silica glass and provides the pathway for light. This core is covered by cladding. When the light reaches the cladding, all of it is reflected back into the core using principle of TIR. Then light will entered in the laser cavity (doped part of laser).

Stimulated emission in the laser cavity

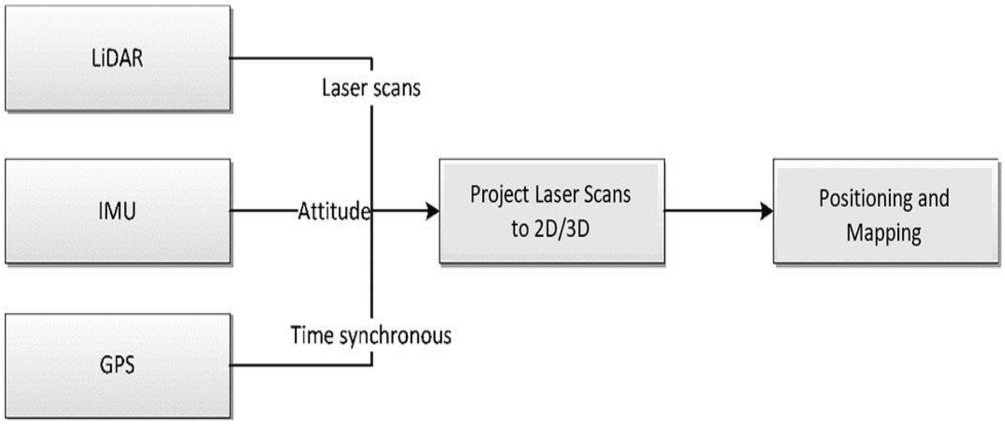
When the laser diode light reaches the doped fibre. it strikes the rare earth element's atoms and excites its electrons to a higher energy level. The number of electrons in the higher energy levels becomes more than lower energy level. This is known as population inversion. When some of these electrons naturally fall to lower energy levels, they emit photons of only a specific wavelength. These photons interact with other excited electrons, stimulating them to emit similar photons and come back to their initial lower energy levels. This is known as stimulated emission. The number of photons identical in wavelength and phase increases. This is known as LASER. (Light Amplification by Stimulated Emission of Radiation).

Amplification of raw laser light into a laser beam

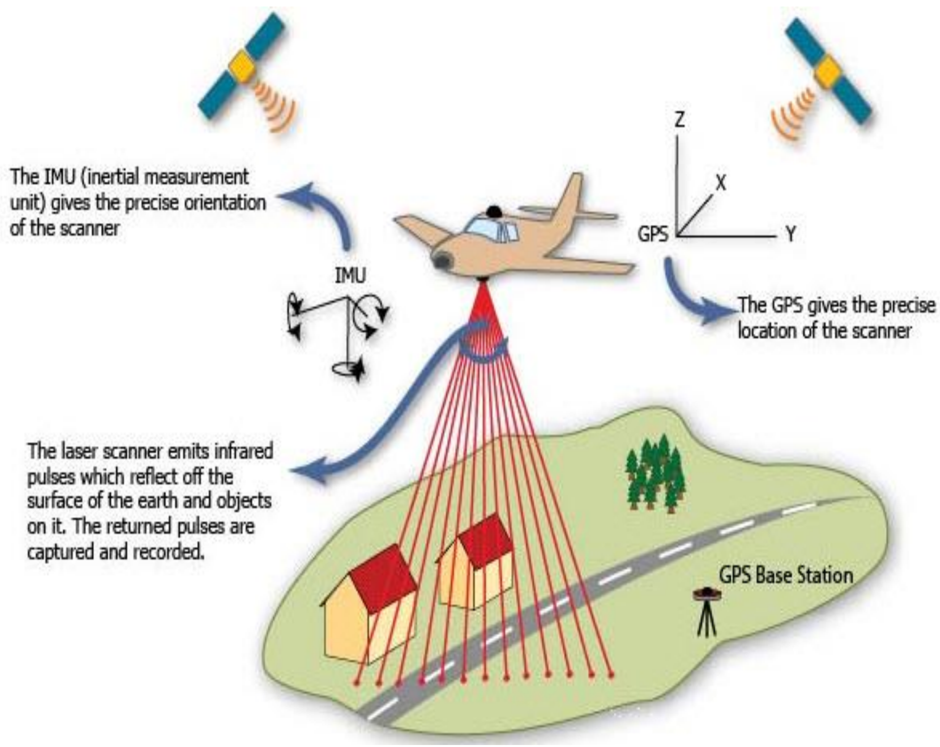
The raw laser light is then passed through the Fibre Bragg Gratings (FBGs of varying reflectivity. The light jumps back and forth between the Bragg Grating. A portion of the laser light passes through in one direction while the remaining light is reflected into the laser cavity. The part that passes through the grating becomes the laser beam. This beam is then sent through an oscillator to improve coherence and then delivered as output.

Because of such a wide range of produced wavelengths, fibre lasers are perfect for a variety of applications such as laser cutting, texturing, cleaning, engraving, drilling, marking and welding. This also enables fibre lasers to find use in many different sectors such as medicine, defence, telecommunications, automotive, spectroscopy, electrical, manufacturing and transportation.

LIDAR



Block Diagram of LiDAR Technology



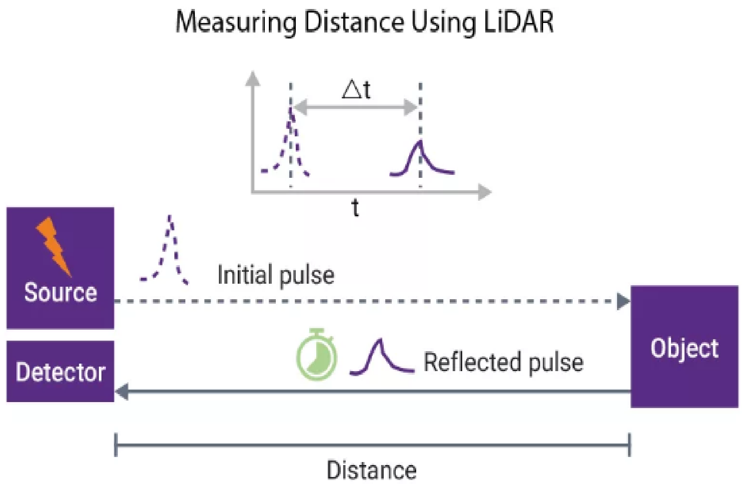
LiDAR stands for Light Detection and Ranging.

The Difference between Radar and LiDAR?

LiDAR and Radar both emit pulses to determine the time it takes to hit a surface and return to the sensor. Radar, however, uses radio waves instead of light pulses. LiDAR technology creates accurate measurements through 3D models, whereas the primary use for Radar is for military purposes i.e. on battleships to detect objects in the vicinity.

LiDAR scanning operating principle

In LiDAR, laser light is sent from a source (transmitter) and reflected from objects. The reflected light is detected by the receiver. The distance between the object and sensor is calculated using the velocity of light, referred to as the Time of Flight (TOF). The time of flight (TOF) is used to develop a distance map of the objects.



The accuracy of LiDAR can vary based on several factors, including the type of LiDAR system, the technology used, the quality of the equipment, and the specific application.

Working of LiDAR

LIDAR system incorporates a laser device, a navigational Inertial Measurement Unit (IMU), a high-precision airborne Global Positioning System (GPS), and a computer interface. The technology uses ultraviolet, visible, or near-infrared light to image objects. The laser emits light pulses and detects the light reflected by the objects. The sensor measures the time between the emission and return of the laser pulse and calculates the distance traveled. Distance traveled is then converted to elevation. These measurements are made using a LiDAR system's key components, including a GPS that identifies the X, Y, Z location of the light energy and an IMU that impart the orientation. This process is also called 'Time of Flight'(ToF) measurement. Contemporary LiDAR systems are sufficiently powerful to fire up to 900,000 pulses every second.

Lidar Formula

The entire process of bouncing a beam of light or laser off an object, receiving the returned signal, and calculating its absolute position in space can be represented mathematically using this formula:

The distance of the object= Speed of Light x Time of Flight/ 2

$$d = c * t / 2$$

A LiDAR system measures the elapsed time it takes for emitted light to travel to the ground and back. The system calculates the distance based on the time delay, creating a point cloud that represents the object‘s shape and position in 3D space. This enables accurate object detection and mapping in various applications. LiDAR has historically been used both on land and in the air.

Types of LiDAR:

Three main types of LiDAR systems - airborne, terrestrial and satellite LiDAR.

Airborne LiDAR is utilised through helicopters or drones for data collection.

Terrestrial LiDAR is installed on moving vehicles or on stationary tripods. These types of LiDAR systems are perfect for modeling and observing static topography.

Satellite or space-borne LIDAR platforms are mounted on satellites that orbit the Earth and tend to cover large areas but with less detail.

Applications of LiDAR

A LiDAR scanner provides ultra-detailed 3D mapping, allowing augmented reality systems to overlay the data on top of a precise and reliable map. A LiDAR created point cloud scanner enhances the accuracy of augmented reality experiences.

LiDAR from Space:

LiDAR systems evolve for several space applications where imaging is needed to identify safe landing sites for vehicles, docking, and driving. LiDAR technology satellites are also used to survey, map, and generate climate prediction models of Earth and other celestial bodies.

LiDAR for Autonomous Cars:

LiDAR technology is a convenient solution to enable obstacle detection, avoidance, and safe navigation through different environments in various vehicles. The technology is sprinkled across many critical automotive and mobility applications, including Advanced Driver Assistance Systems (ADAS) and Autonomous driving.

LiDAR and IoT:

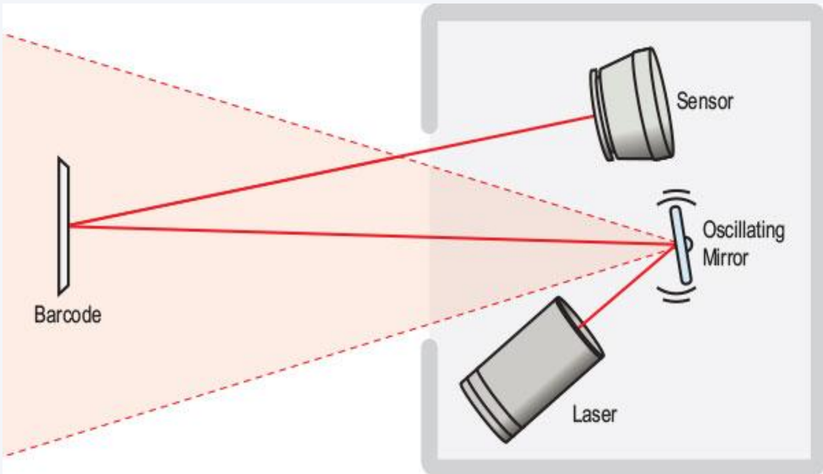
A few facets of LiDAR make it particularly useful in specific Internet of Things (IoT) applications. LiDAR solutions play a pivotal role in delivering on the IoT's promise – increasing safety, productivity and efficiency across a wide variety of applications for smart cities, infrastructure, agriculture, medical, retail and beyond

LiDAR for 3D Printing:

It is already possible to create 3D-printed objects through photographic modeling. With LiDAR, much more detailed modeling data can be abstracted for even more interesting 3D printing projects.

Barcode Scanners

Laser barcode scanner



A barcode is pattern of black and white bars of varying widths used to represent a specific set of numbers that identify products.

There are two types of barcodes:

Linear Barcode - can hold any type of text (written) information

The most common one that we see is called the UPC or the Universal Product Code

It is made up of two parts: the barcode and a 12 digit number.

2D type of barcode - It is less common and it can store more complex information, such as price, quantity,

Each type of Barcode requires a specific barcode scanner

Barcode scanner is a handheld device that consists of many parts to read the information on a barcode.

How does a Barcode Scanner Work?

1. The scanner emits an LED light or a laser onto the barcode.
2. A light is the reflected off an optical lens into a light-detecting electronic component called a photoelectric cell.
3. The white areas reflect the least light while the black reflect the most.
4. The difference in light reflected allows the barcode to identify the unique set of numbers
5. As the scanner moves along the barcode, the cell generates a pattern of on-off pulses that correspond to the black and white lines.
6. A black line generates an "off" pulse and a white line generates an "on" pulse.
7. An electric circuit within the scanner converts these pulses to binary digits (zero or one)
8. These digits are the sent to the tracking computer which detects those numbers and stores the information contained within the code

Industrial application of Laser:

Laser systems are used in many manufacturing processes. Industrial lasers are used to

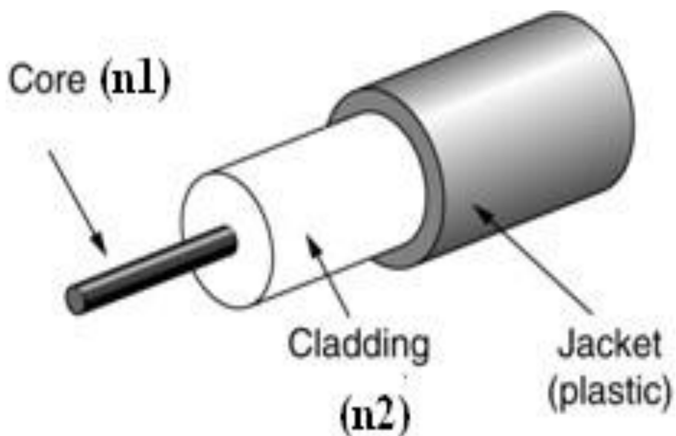
1) To cut metals and fabrics.
2) Mark tracking code for industrial traceability.
3) Weld metals with high precision.
4) Clean metal surface.
5) Change the surface roughness.
6) Measure part dimensions.

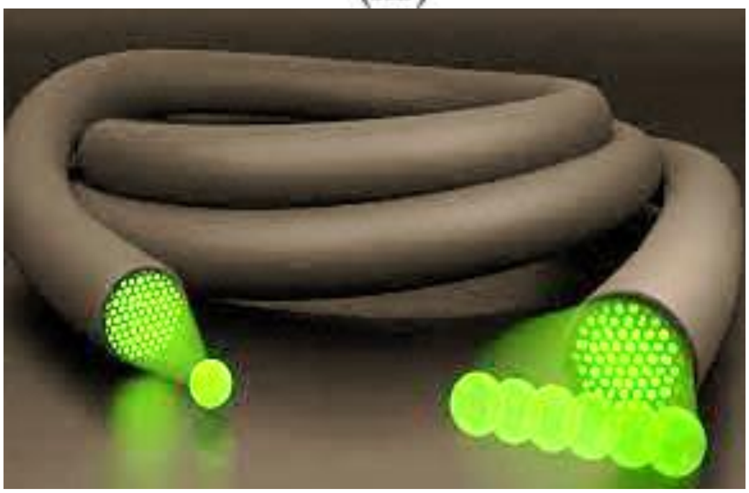
Fiber Optics Communication

A communication system is required to exchange the information (audio, video, text, and data) from one place to another place over a long distance. The conventional communication systems are wire communication, radio and microwave communication, satellite communication.

The fiber optics communication is a communication system in which information in the form of light is used to send over a long distance using optical fiber. Information is first converted in to proportional light waves and sends through the optical fiber from transmitter to receiver.

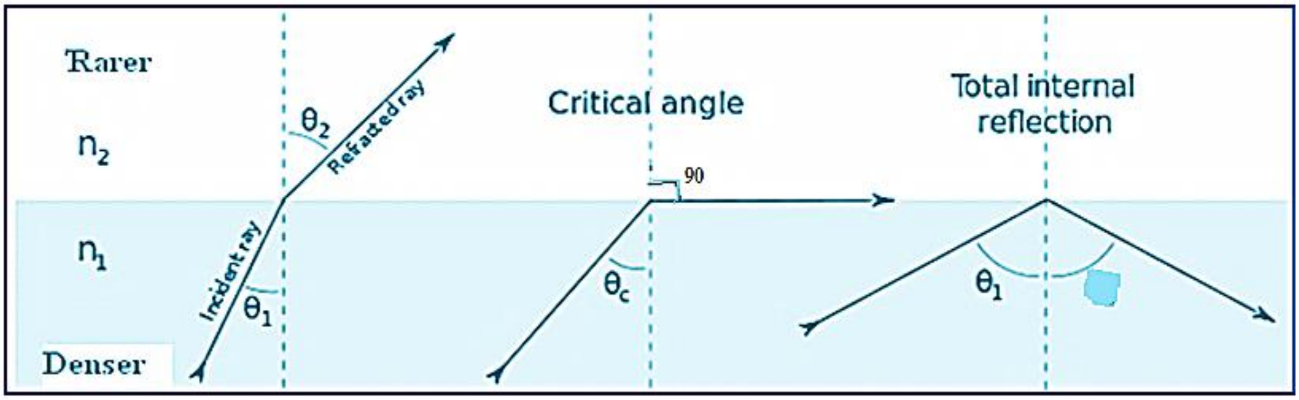
An optical fiber is a hair thin narrow tube of transparent material (like silica), having uniform refractive index and uniform diameter throughout its entire length known as core. The core is covered by another material of slightly lower refractive index known as cladding. The cladding is covered by insulating material like plastic to protect the fiber known as buffer coating or jacket. The principle of total internal reflection (TIR) is used to transfer the information in the form of light through the optical fiber.





Q. Explain the principle of TIR used in fiber optics communication?

Principle of Total Internal Reflection (TIR):-



$\theta_1 = \theta c$ $\theta_1 > \theta c$

1) Consider the light waves incident from denser medium of refractive index n_1 to the rarer medium of refractive index n_2.
2) If θ1 is angle of incidence θ 2 is angle of refraction, then by Snell's law of refraction

   $n_1 \sin \theta 1 = n_2 \sin \theta 2$
3) **I**f angle of incidence increases, angle of refraction also increases and at particular angle of incidence, angle of refraction becomes 90^o. This angle of incidence is known as critical angle of incidence θc.

$$n_1 \sin \theta c = n_2 \sin 90^o$$

$$\sin\theta_c = \frac{n_2}{n_1}$$

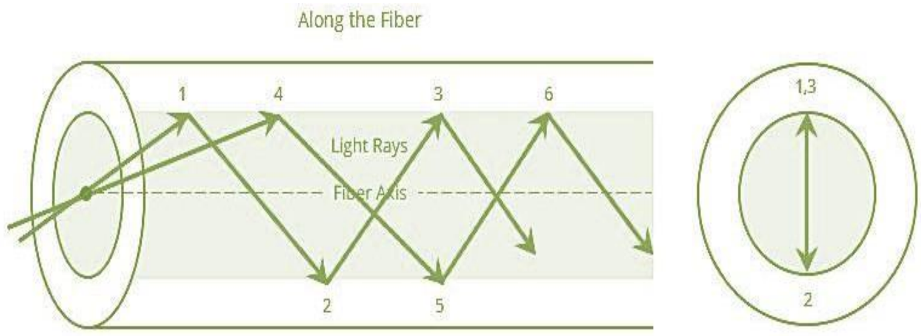
$$\theta_c = \sin^{-1}(n_2/n_1)$$

4) If the angle of incidence is greater than critical angle θc, then incident light waves did not refract in the rarer medium but reflected back in the same denser medium. This reflection is known as total internal reflection.
5) **Thus, for TIR there are two conditions**

   a. Light should be launched through the denser medium.

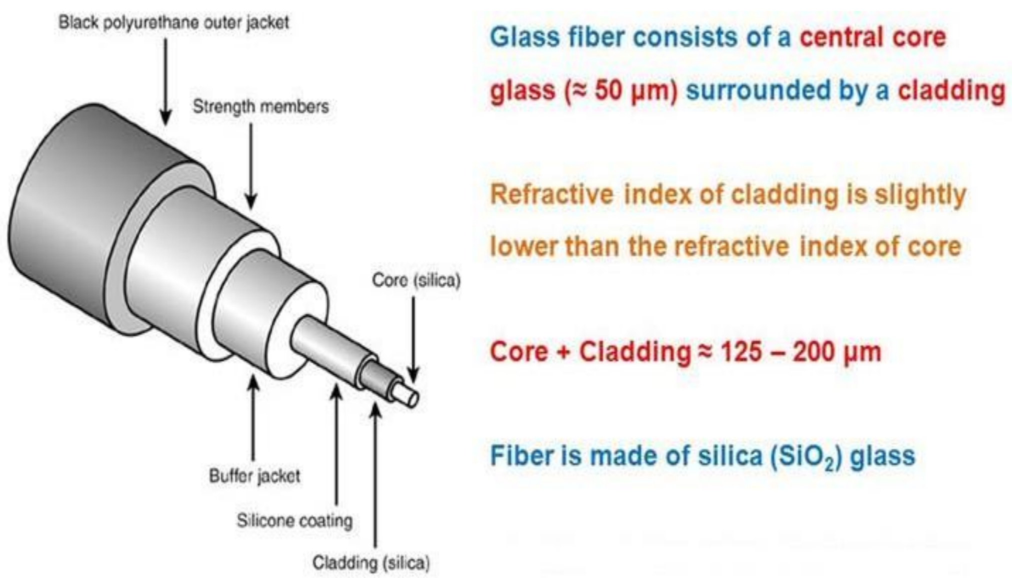
   b. The angle of incidence must be greater than critical angle of incidence.

Optical fiber cable: -



The optical fibers are classified in to different types according material used, refractive index profile and modes of propagation. The structure of optical fiber cable is as shown in figure.

In fiber optics communication, the input Information is first converted in to proportional light waves and sends through the optical fiber from transmitter to receiver. The incident light travels along the entire length of the optical fiber by multiple reflections with in core using principle of TIR and exists from the other end.



Classification of optical fiber:

According to material used:

1) Glass fiber. 2) Plastic

fiber. According to modes of

propagation:

1) Single mode optical fiber. 2) Multimode optical

fiber. According to refractive index profile:

1) Step index optical fiber. 2) Graded index optical fiber.

Glass fiber:

The glass fibers are made from material silica of refractive index 1.458. The cladding is made from pure silica and silica doped with germanium or phosphorous is used for core.

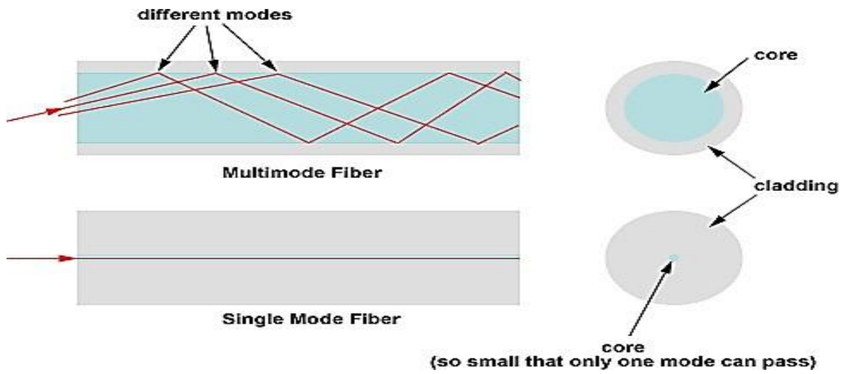
Plastic fiber:

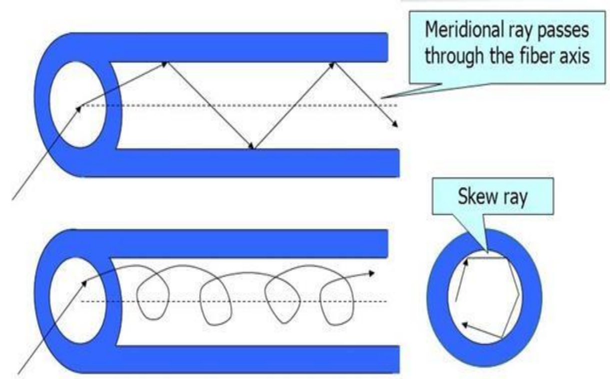
In plastic fiber polystyrene is used for core and silicon resin is used for cladding.

Q. What is mode of propagation? Distinguish between single mode & multimode propagation?

Mode of propagation:

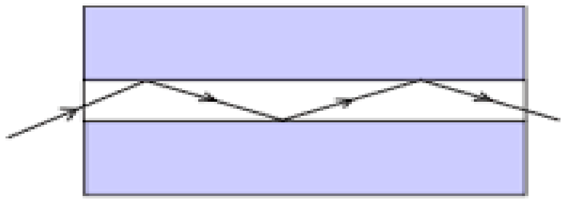
- When the light waves are incident on the fiber, the fiber will not accept all the incident light waves for propagation. Only selected waves of particular wavelengths are accepted by the fiber for propagation.
- Depending upon the ratio of diameter of core to the wavelength of light wave (d/λ), the fiber accepts only some selected wavelengths of incident light for efficient transmission using principle of TIR known as modes of propagation.
- Depending upon mode of propagation optical fibers are classified as single mode and multimode optical fiber.
- The light waves travelling along the axis of the fiber in straight line is axial mode and light waves travelling along zig-zag path is non-axial mode.
- The light waves travelling along zig-zag path crossing the fiber axis are known as meridonal rays.
- The light waves travelling along zig-zag path but not crossing the fiber axis are known as skew rays.





Skew ray follows helical path in optical fiber

Single mode fiber:

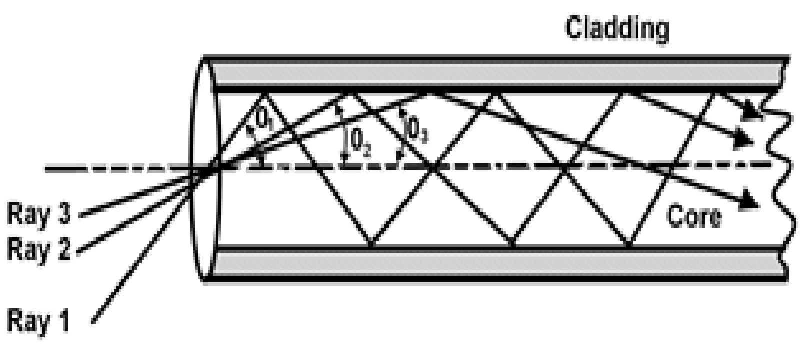


Ray propagation

Support only one mode of propagation.

1) Core diameter is very small.
2) Laser diode is used as light source.
3) Bandwidth is high.
4) Transmission losses are less.
5) Used for long distance communication.
6) E.g. Step index fiber of small core diameter.

Multimode optical fiber:

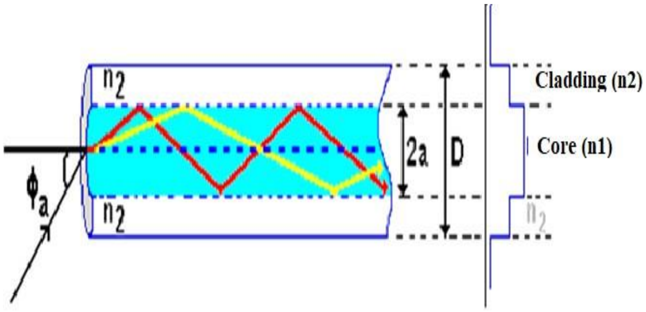


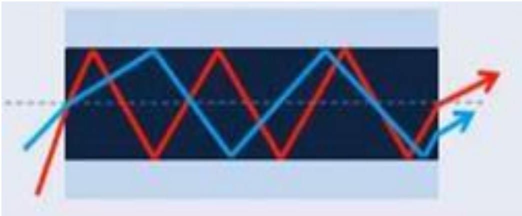
1) Support large number of modes of propagation.
2) Core diameter is large.
3) LED is used as light source.
4) Bandwidth is small.
5) Transmission losses are more.
6) Used for short distance communication.
7) E.g. Step index fiber of large core diameter and graded index fiber.

Q. Explain step index & graded index optical fiber?

Q. Distinguish between Step index & Graded index optical fiber?

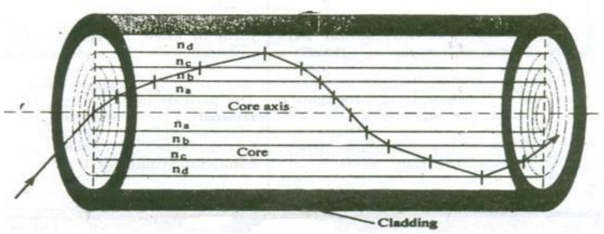
Step index fiber:

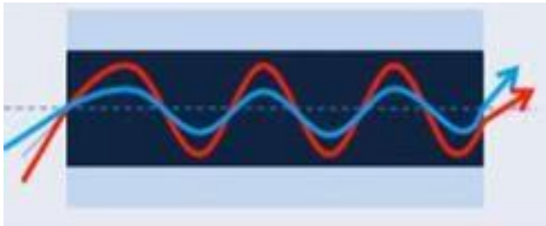




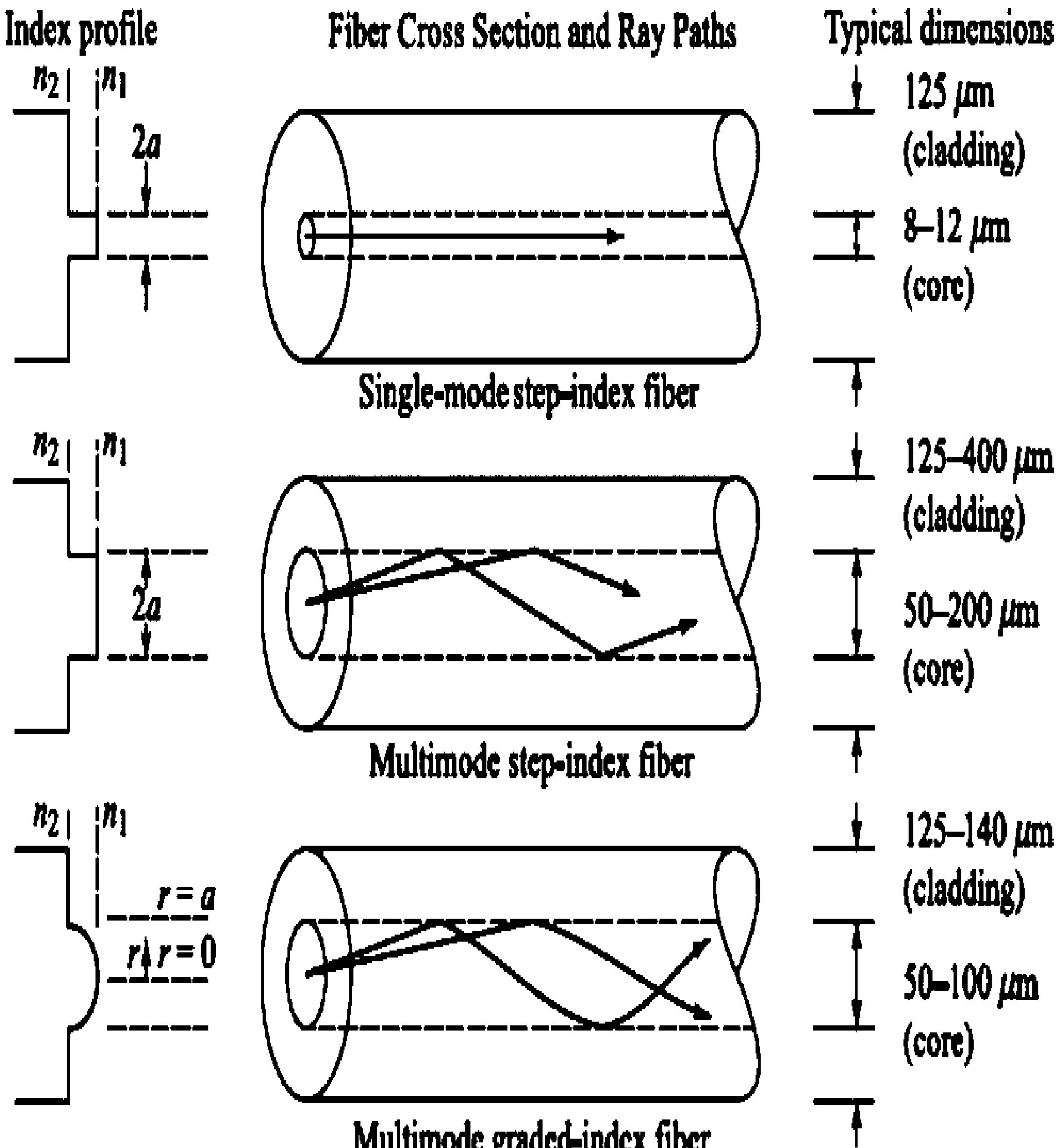
1) Consist of core of uniform diameter and refractive index and cladding of slightly less R.I. than core.
2) Light rays are travel along zig-zag triangular path.
3) All light rays travel with same speed along different path and arrive at the other end at different time, therefore signal distortion takes place.
4) Bandwidth is small.
5) Numerical aperture is high.
6) Attenuation is higher.
7) They can be single mode or multimode depending upon core diameter.

Graded index fiber:





1) No core and cladding separately, but fiber is made with a material of varying R.I. The R.I. is maximum along the axis and decreases away from the axis along the diameter (na>nb>nc>nd).
2) Light rays are travel along sinusoidal path.
3) All light rays are travel with different speed having different degree of curvature and will arrive at the other end at same time; therefore, signal distortion not takes place.
4) Bandwidth is high.
5) Numerical aperture is small.
6) Attenuation is lower.
7) They are only multimode.

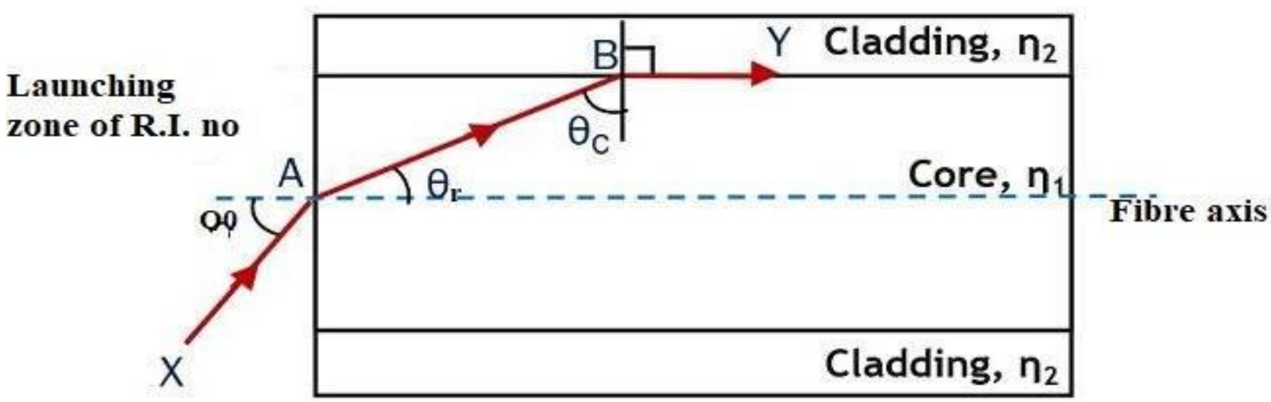

Index profile
Fiber Cross Section and Ray Paths
Typical dimensions
n_2
n_1
2a
125 μm
(cladding)
8–12 μm
(core)
Single-mode step-index fiber
n_2
n_1
2a
125–400 μm
(cladding)
50–200 μm
(core)
Multimode step-index fiber
n_2
n_1
r = a
r
r = 0
125–140 μm
(cladding)
50–100 μm
(core)
Multimode graded-index fiber

Q. What is numerical aperture and acceptance angle of optical fiber? Derive its expression for step index optical fiber?

Q. Explain the following terms in case of optical fiber 1) Numerical aperture 2) Acceptance angle. 3) Normalized frequency 4) Modes of propagation.

Numerical Aperture (N.A.) of optical fiber:

Numerical aperture is light gathering capacity of optical fiber. The ability of optical fiber to accept the light and to guide the light efficiently throughout its entire length using the principle of TIR is known as numerical aperture (N.A.). Numerical aperture is always less than 1 for total internal reflection of light.



Consider step index optical fiber of core radius r. If n_1 – R.I. index of core.

n_2 – R.I of cladding

n_0 – R.I of launching zone

θ_0 – angle of incidence with fiber axis known as acceptance angle.

θ_r – angle of refraction.

θ_c – Critical angle of incidence at core- cladding interface.

At air-glass boundary (point A) by Snell's law of refraction,

$$n_0 \sin \theta_0 = n_1 \sin \theta_r \text{-----(1)}$$

The maximum angle of incidence made by the incident light waves with fiber axis is known as acceptance angle $\boldsymbol{\theta_0}$.

Sine of the acceptance angle $\boldsymbol{\theta_0}$ is also known as numerical aperture of optical fiber.

$\sin\theta_0$ is the N.A. of optical fiber.

At core-cladding interface (point B), if θc is critical angle of incidence, then angle of refraction is 90^0. $n_1 \sin\theta c = n_2 \sin 90^0$ (2)

$n_1 \sin\theta c = n_2$ since $\sin 90^0 = 1$

$$\sin\theta_c = \frac{n_2}{n_1}$$

From fig. angle of refraction $\theta r = 90^0 - \theta c$ Therefore equation (1) => $n_0 \sin\theta_0 = n_1 \sin(90^0 - \theta c)$

$$n_0 \sin\theta_0 = n_1 \cos\theta c$$

$$n_0 \sin\theta_0 = n_1\sqrt{1 - sin^2\theta c}$$

$$n_0 \sin\theta_0 = n_1\sqrt{1 - \left(\frac{n2}{n1}\right)^2}$$

$$n_0 \sin\theta_0 = \sqrt{n1^2 - n2^2}$$

$$\sin\theta_0 = \frac{\sqrt{n1^2 - n2^2}}{n0}$$

$$\textbf{NA} = \frac{\sqrt{n1^2 - n2^2}}{n0}$$

If light wave incident from air medium, $n_0 = 1$

$$\textbf{Sin}\ \theta_0 = \sqrt{n1^2 - n2^2}$$

$$NA = \sqrt{n_1^2 - n_2^2}$$

The maximum angle of incidence made by the incident light waves with fiber axis is known as acceptance angle.

$$\textbf{Acceptance angle}\ \theta_0 = \sin^{-1}\sqrt{n1^2 - n2^2}$$

$$\theta_0 = \sin^{-}(N.A)$$

N.A. in terms of fractional refractive index difference (△)

$$NA = \sqrt{n_1^2 - n_2^2}$$

$$\textbf{N.A} = \sqrt{(n1 - n2)(n1 + n2)}$$

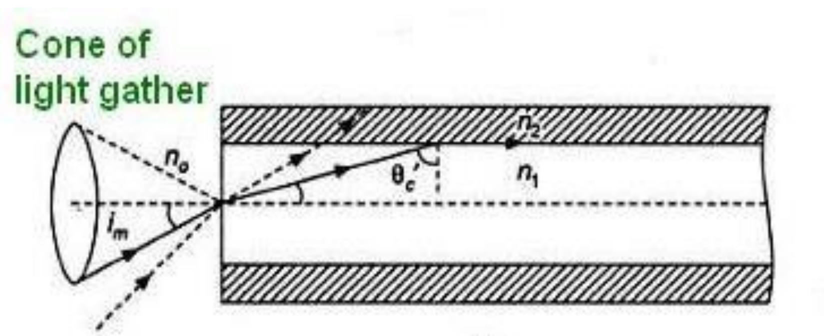
But $n_1 \approx n_2$ therefore $n_1 + n_2 = 2n1^2$

$$\text{N.A} = \sqrt{2n1\,(n1 - n2)}$$

$$N.A = \sqrt{\frac{2n1^2\,(n1-n2)}{n1}}$$

$$N.A = n1\sqrt{2\,\Delta}$$

Where $\Delta = \frac{n1-n2}{n1}$ is known as fractional refractive index difference.



V - Number or normalized frequency:

An optical fiber is characterized by another parameter called as the V- number or normalized frequency. It is given by

$$V = \frac{2\pi a}{\lambda}\sqrt{n1^2 - n2^2}$$

$$V = \frac{2\pi a}{\lambda}\,N.A$$

Where 'a' is core radius and λ is wavelength of light.

When V = 2.405, then λ is known as cutoff wavelength of the optical fiber.

Hence cutoff wavelength is $\lambda_{cutoff} = \frac{2\pi a}{2.405}\,N.A$

For single mode propagation V <

2.405 For multimode propagation V

> 2.405

The maximum number of modes the fiber can support is given by

$$N_m = \frac{V^2}{2}$$ **for step index fiber.**

$$N_m = \frac{V^2}{4}$$ **for graded index fiber.**

Q. Write the advantages of fiber optics communication system?

Advantages of communication through optical fiber:

1. Electromagnetic interference is avoided.
2. High security against tapping.
3. No short circuit problems.
4. Less expensive & saves copper.
5. Due to high information capacity multiple channel routes may be compressed onto much smaller cables.
6. Optical fiber has ability to accept & transport the light at large angles without loss of intensity.
7. Optical fibers are flexible. Hence it is possible to view the area which is not observed directly.
8. Fiber optics system has a large bandwidth.
9. The signal leakage is nil due to the total internal reflection& hence there is no cross – talk.
10. It gives nearly proof communication during the wartime.

Q. Explain the different applications of optical fiber?

APPLICATIONS: -

1) **Fiber optics delay lines: -**The fiber optic cables are widely used in electronic field to produce required delay.

2) **Fiber optics sensors: -** It is used as a sensor for electric field, magnetic field, temperature, mechanical force etc.

3) **Fiber endoscope: -** It is used to study the interior of the lungs & the other parts of the human body that cannot be viewed directly. It can also be used to study tissues & blood vessels far below the skin. The equipment used in medical field for this purpose which consist of bundle of flexible fiber is known as endoscope.

4) **Optical fibers are used in computers: -** Exchange the information between different terminals in a network.

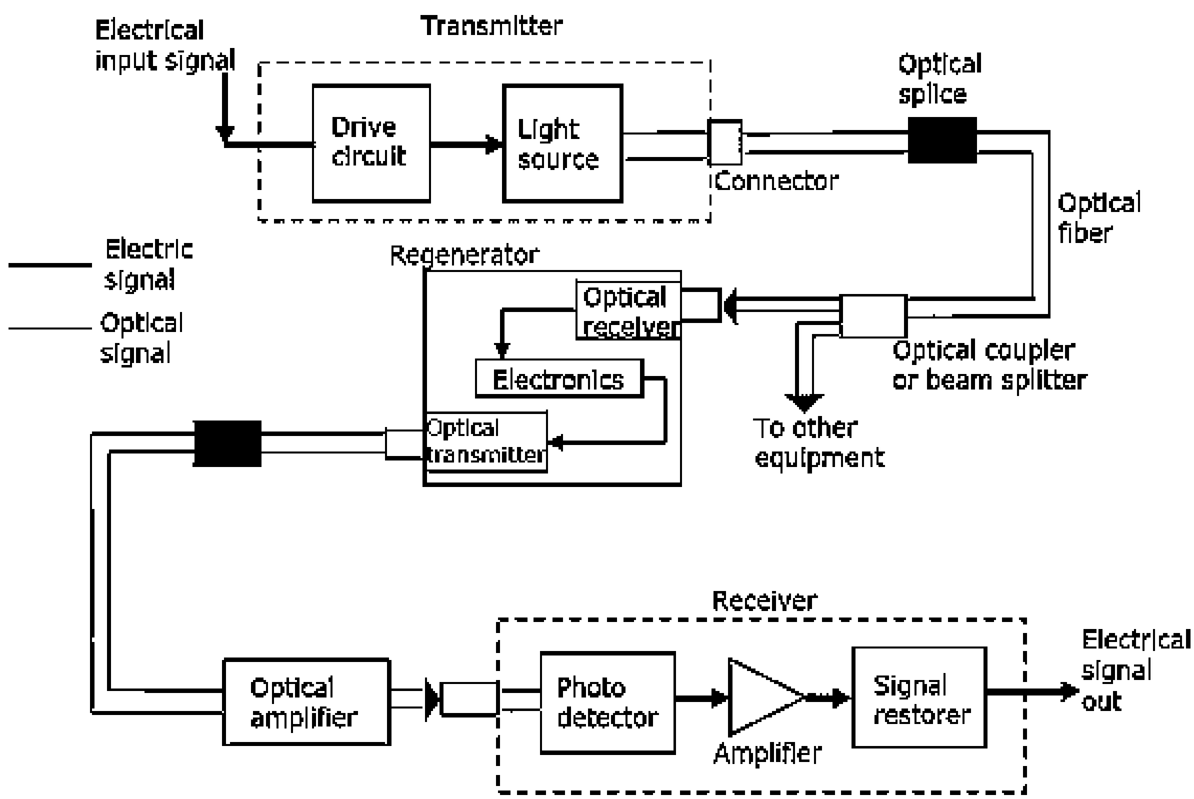
5) The optical fibers are used to exchange the information in cable, space vehicles & submarine etc.

Applications of Fiber Optics communication system:

- Used in telephone systems
- Used in sub-marine cable networks
- Used in data link for computer networks, CATV Systems
- Used in CCTV surveillance cameras
- Used for connecting fire, police, and other emergency services.
- Used in hospitals, schools, and traffic management systems.
- They have many industrial uses

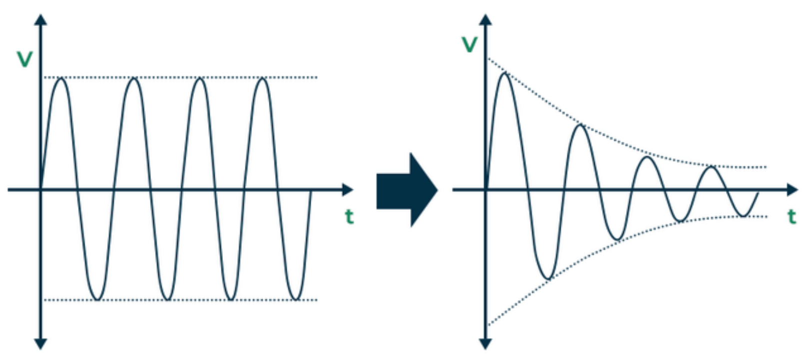
Q. Draw the block diagram of fiber optics communication system & explain the function of each block?

FIBER OPTICS COMMUNICATION SYSTEM: -



1) Fiber optics communication system provides large bandwidth, high speed as well as security to exchange the information over a long distance. The information is exchange using light wave by optical fiber.
2) The information in any form (audio, video, text, image or data) is first converted in to proportional electrical signal using electrical data generator.
3) The electrical signal is given to light source through driver circuit. The driver circuit produces the electrical signal suitable to drive the light source.
4) The LED or semiconductor laser diode is used as light source to convert the information in to proportional light wave.
5) The modulated light wave containing information is efficiently guided using optical fiber over a long-distance using principle of TIR.
6) At receiver this light is detected by using photo diode or photo transistor. The light detector converts the information again in to electrical signal.
7) This electrical signal is amplified, restore and given to the output device. The out device may be speaker, printer, display etc. depending upon type of information.
8) For long distance communication repeaters are provided along with optical fiber cable. But repeaters required are less in number than wire communication system.

Attenuation in Optical Fiber:



Optical fiber are used to transmit signals using light over large distances. Attenuation in optical fibers occurs when the light intensity is reduced as it propagates through the fiber. It is a type of optical loss and it limits the distance over which it can travel.

Types of Attenuation in Optical Fiber

There are two types of attenuation in optical fiber,

- Intrinsic Attenuation
- Extrinsic Attenuation

Intrinsic Attenuation

Intrinsic attenuation happens due to absorption and scattering.

- **Absorption** – It happens due to the imperfections in the optical fiber. It is caused by the atomic defects in the glass components. When light passes through fiber it may be absorbed by one or more components of glass. Absorption loss can happen from transition metal element impurities. This loss value is generally 0.4 dB/km at 1310nm and 0.25 dB/km at 1550nm.
- **Scattering** – It happens due to fiber materials and structural imperfections. Fiber structures have regions of high and low molecular density. Also several oxides present in glass leads to compositional fluctuations. This scattering of lights lead to loss of power or attenuation. Rayleigh scattering arises from variation in the density of fiber materials leading to random fluctuations.

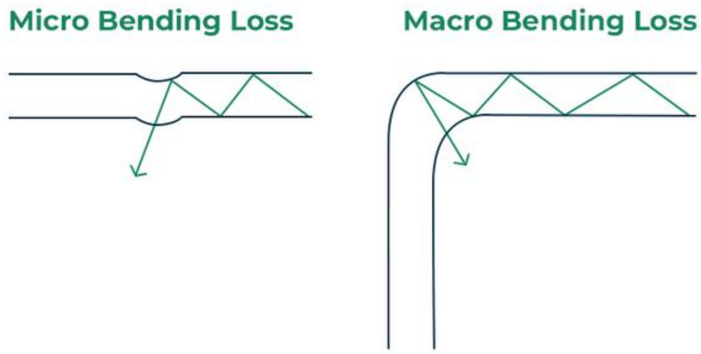
Extrinsic Attenuation

Extrinsic attenuation is the loss that is caused by external factors. It happens from Radiative loss (bending loss) and coupling loss (splicing and connector loss).

- **Radioactive Losses**

It occurs when a fiber undergoes a bend. Fiber bends are of two type's macro bends and micro bends.

- Macro bending loss happens if the radius of the core is larger than the fiber diameter then it may cause large curvature at the positions where the fiber is bend and light will escape.
- Micro bending loss happens due repetitive small scale fluctuations caused by non-uniformities inside the fiber which appears due to non-uniform pressure during cabling or during manufacturing.

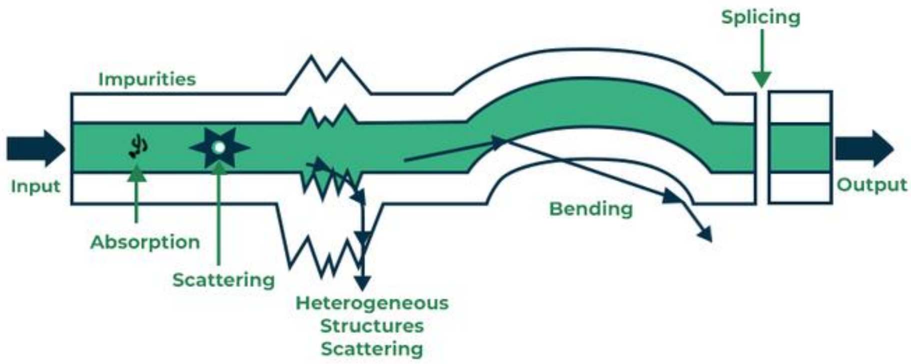


Bending Losses

Coupling Losses

- Fiber optic splicing also causes losses. By joining two optical fibers end to end splicing aims to ensure that the light passing through it is almost as strong as the original wire without any joins. Splicing loss value is generally 0.05dB to 0.3dB.

- Connector losses or insertion losses in optical fiber results from the insertion of a device in a transmission line or optical fiber. This loss is generally around 0.3dB to 0.75dB.



Losses in Optical Fiber

Measurement of Attenuation in Optical Fiber and Attenuation Coefficient

The attenuation coefficient is an important parameter of optical fiber which is used to measure the rate at which the intensity of light decreases. It is denoted by ∝ (alpha).

The attenuation coefficient (α) is mathematically represented as:

$$\text{Number of decibels} = 10\log_{10}(\frac{P_i}{P_o})$$

Power P(L) at a distance L is given as,

$$P(L) = P(O)e^{-\propto L}$$

Where P(0) is power at origin

Then attenuation coefficient of fiber in decibel per kilometer is,

$$\propto = \left(\frac{10}{L}\right)\log(\frac{PO}{PL})$$

Where,

- ∝ is the attenuation coefficient (in dB/km)
- L is the length of the fiber over which the measurement is taken
- P(0) is the initial optical power
- P(L) is the optical power at distance L from input

Higher the attenuation coefficient higher is the loss while lower value means lower loss and a high quality fiber.

Calculate the numerical aperture of a fibre with core index = 1.65 and cladding index = 1.53.

Given : $\mu_1 = 1.65$ and $\mu_2 = 1.53$

Numerical aperture = $\sqrt{(\mu_1)^2 - (\mu_2)^2} = \sqrt{(1.65)^2 - (1.53)^2} = 0.618$

Calculate the acceptance angle for an optical fibre whose core refractive index is 1.48 and cladding refractive index is 1.39.

Given : $\mu_1 = 1.48$ and $\mu_2 = 1.39$

Numerical aperture = $\sqrt{(\mu_1)^2 - (\mu_2)^2} = \sqrt{(1.48)^2 - (1.39)^2} = 0.508$

Acceptance Angle = $\sin^{-1}(\mathrm{N.A.}) = \sin^{-1}(0.508) = 30.53^0$

The numerical aperture of an optical fibre is 0.5 and core refractive index is 1.54. Find refractive index of cladding.

Given : $\mu_1 = 1.54$ and $N.A. = 0.5$

Numerical aperture = $\sqrt{(\mu_1)^2 - (\mu_2)^2}$

$\therefore \mu_2 = \sqrt{(\mu_1)^2 - (N.A.)^2} = \sqrt{(1.54)^2 - (0.5)^2} = 1.456$

Numerical aperture of a fibre is 0.5 and core refractive index is 1.48. Find cladding refractive index and acceptance angle.

Given : $\mu_1 = 1.48$ and $N.A. = 0.5$

Numerical aperture = $\sqrt{(\mu_1)^2 - (\mu_2)^2}$

$\therefore \mu_2 = \sqrt{(\mu_1)^2 - (N.A.)^2} = \sqrt{(1.48)^2 - (0.5)^2} = 1.393$

Acceptance Angle = $\sin^{-1}(\mathrm{N.A.}) = \sin^{-1}(0.5) = 30^0$

A step index fibre is made with a core of index 1.52 and diameter 29 μm and cladding refractive index 1.5189. If it is operated at wavelength 1.3 μm, find V number of fibre and no. of mode it will support.

Given : $\mu_1 = 1.52$ and $\mu_2 = 1.5189, d = 29 \times 10^{-6}m,$

$\lambda = 1.3 \times 10^{-6}m$

$$V = \frac{2\pi a}{\lambda}\sqrt{(\mu_1)^2 - (\mu_2)^2} = \frac{\pi d}{\lambda}\sqrt{(\mu_1)^2 - (\mu_2)^2}$$

$$= \frac{3.142 \times 29 \times 10^{-6}}{1.3 \times 10^{-6}}\sqrt{(1.52)^2 - (1.5189)^2} = 4.05$$

As $V > 2.405$, $N_m \approx \frac{1}{2}V^2 = \frac{1}{2}(4.05)^2 = 8.20$

Thus Fibre will support 8 modes

Find the core radius necessary for single mode operation at 850 nm in step index fibre with n_1 =1.480 and n_2 = 1.47. What is the numerical aperture and maximum acceptance angle of this fibre.

Given : $n_1 = 1.480$ and $n_2 = 1.47$, $\lambda = 850 \times 10^{-9}m$

For Single mode operation , $V \le 2.405$

$$V = \frac{2\pi a}{\lambda}\sqrt{(n_1)^2 - (n_2)^2}$$ where a is radius of core

$$\therefore a = \frac{V\lambda}{2\pi\sqrt{(n_1)^2 - (n_2)^2}} = \frac{2.405 \times 850 \times 10^{-9}}{2 \times 3.142 \times \sqrt{(1.48)^2 - (1.47)^2}} = 1.894 \times 10^{-6}m$$

Numerical Aperture= $\sqrt{(n_1)^2 - (n_2)^2} = \sqrt{(1.48)^2 - (1.47)^2} = 0.1717$

Acceptance Angle $= \sin^{-1}(N.A.) = \sin^{-1}(0.1717) = 9.887^0 = 9^0 53'$

An optical fibre has core diameter of 6 micrometer and core refractive index is 1.45. The critical angle is 87°. Calculate i) refractive index of cladding, ii) acceptance angle and iii)number of modes propagating through fibre when wavelength of light is 1 micrometer

Given : $\mu_1 = 1.45,\ \mu_2 = ?,\ d = 6 \times 10^{-6}\ m,\ \phi_c = 87^0,\ \lambda = 1 \times 10^{-6}\ m$

As $\sin\phi_c = \frac{\mu_2}{\mu_1}$, $\mu_2 = \mu_1 \sin\phi_c = 1.45 \times \sin(87) = 1.448$

$$\text{Acceptance Angle} = \sin^{-1}(N.A.) = \sin^{-1}\left(\sqrt{(\mu_1)^2 - (\mu_2)^2}\right)$$

$$= \sin^{-1}\left(\sqrt{(1.45)^2 - (1.448)^2}\right) = 4.366^0$$

$$V = \frac{2\pi a}{\lambda}\sqrt{(\mu_1)^2 - (\mu_2)^2} = \frac{\pi d}{\lambda}\sqrt{(\mu_1)^2 - (\mu_2)^2}$$

$$= \frac{3.142 \times 6 \times 10^{-6}}{1 \times 10^{-6}}\sqrt{(1.45)^2 - (1.448)^2} = 1.435$$

As V number is less than 2.405, Number of Modes propagating is 1. It is a single mode fibre

Q1. Calculate the numerical aperture and hence the acceptance angle for an optical fiber. Given that the refractive indices of the core and the cladding are 1.45 and 1.40 respectively.

Given:- $\mu_1 = 1.45$; $\mu_2 = 1.40$

Formula :- $N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$

Solution:- N.A.= $\sqrt{1.45^2 - 1.40^2}$

= 0.3775

$\theta_0 = \sin^{-1}(N.A.) = \sin^{-1}(0.1425)$

= 8.192°

Ans:- N.A. of fiber is 0.3775 and its acceptance angle is 8.192°

Q2. The refractive index of core and cladding of a SI fiber are 1.52 and 1.41 respectively. Calculate (i) critical angle (ii) NA and (iii) maximum incidence angle.

Given:- $\mu_1 = 1.52; \quad \mu_2 = 1.41$

Formula:- $\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1}$; $N.A. = \sin\theta_0 = \sqrt{\mu_1^2 - \mu_2^2}$

Solution:- $\varphi_c = \sin^{-1}(\frac{1.41}{1.52}) = 68.06^\circ$

$N.A. = \sqrt{1.52^2 - 1.41^2} = 0.5677$

$\theta_0 = \sin^{-1}(0.5677)$

$= 34.59^\circ$

Ans:- The critical angle is 68.06° and N.A. is 0.5677 and θ_0 is 34.59°

Q3. An optical fiber has a NA of 0.20 and refractive index of cladding is 1.59. Determine the core refractive index and the acceptance angle for the fiber in water which has a refractive index of 1.33.

Given:- NA=0.20; μ_2=1.59; μ_o=1.33

Formula:- $N.A. = \sin\theta_0 = \frac{\sqrt{\mu_1^2 - \mu_2^2}}{\mu_o}$

Solution:- $N.A. = \sqrt{{\mu_1}^2 - {\mu_2}^2}$

$\mu_1 = \sqrt{N.A.^2 + {\mu_2}^2} = \sqrt{0.2^2 + 1.59^2} = 1.6025$

$\theta_0 = \sin^{-1}\frac{N.A.}{\mu_o} = 8.64^\circ$

Ans:- The R.I. of core is 1.6025 and acceptance angle 8.64°

Q4. A typical relative refractive index difference for an optical fiber is 1%. Estimated the numerical aperture and the critical angle at the core cladding interface if the core refractive index is 1.46.

Given:- Δ=0.01; μ_1=1.46

Formula:- $\Delta=\frac{\mu_1-\mu_2}{\mu_1}$; N.A.= $\mu_1\sqrt{2\Delta}$; $\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1}$

Solution:- N.A.=1.46(2 x 0.01)$^{1/2}$ = 0.2064

$\Delta = 1 - \frac{\mu_2}{\mu_1}$

$\frac{\mu_2}{\mu_1} = 1 - \Delta = 0.99$

$\varphi_c = \sin^{-1}0.99 = 81.89°$

Ans:- The N.A. is 0.2064 and the critical angle is 81.89°.

Q5. A glass clad fiber is made with core glass of refractive index 1.5 and the cladding is doped to get a frictional index difference of 0.0005. Find [a] the refractive index of the cladding. [b] the critical internal reflection angle [c] Acceptance angle.

Given :- μ_1=1.5; Δ= 0.0005

Formula:- $\Delta = 1 - \frac{\mu_2}{\mu_1}$; N.A.= $\sqrt{\mu_1^2 - \mu_2^2}$

Solution:- $\mu_2 = \mu_1(1-\Delta) = 1.5(1-0.0005) = 1.49925$

$\varphi_c = \sin^{-1}\frac{\mu_2}{\mu_1} = \sin^{-1}\frac{1.49925}{1.5} = 88.18°$

$\theta_0 = \sin^{-1}\sqrt{\mu_1^2 - \mu_2^2}$

$= \sin^{-1}\sqrt{1.5^2 - 1.49925^2} = 2.718°$

Ans:- The R.I of cladding is 1.49925 and the critical angle is 88.18° and the acceptance angle is 2.718°.

Q6. A step index fiber has core diameter 29×10^{-6}m . The refractive indices of the core and the cladding are 1.52 and 1.5189 resp. If the light of wavelength 1.3μm is transmitted through the fiber, determine [a]Normalized frequency of the fiber. [b] The number of modes fiber will support.

Given :- $d = 29 \times 10^{-6} m$; $\lambda = 1.3 \times 10^{-6} m$; $\mu_1 = 1.52$; $\mu_2 = 1.5189$.

Formula :- $V = \frac{2\pi r \sqrt{\mu_1^2 - \mu_2^2}}{\lambda}$; $N_m = \frac{V^2}{2}$

Solution:- $V = \frac{3.14 \times 29 \times 10^{-6}}{1.3 \times 10^{-6}} \times \sqrt{(1.5)^2 - (1.5189)^2} = 4.049$

$$N_m = \frac{V^2}{2} = \frac{(4.049)^2}{2} = 8.197$$

Ans:- Normalized frequency is 4.049 and the number of modes is 8.

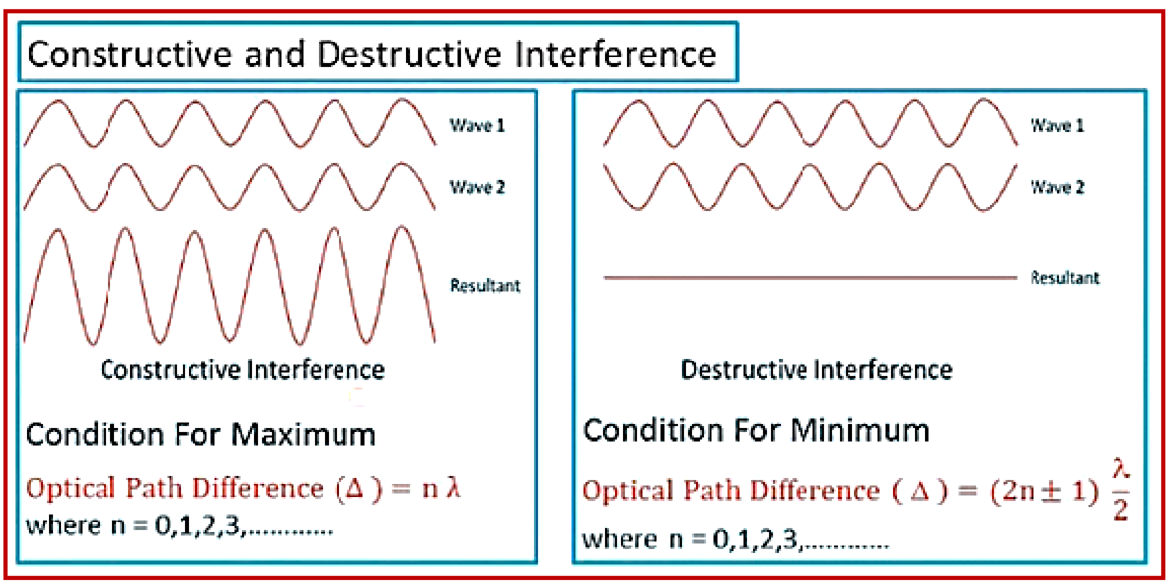
OPTICS (Interference of light)

(Prerequisites: Wave front and Huygens's principle, reflection and refraction, Interference by division of wave front, Young's double slit experiment)

Interference by division of amplitude, Interference in thin film of constant thickness due to reflected and transmitted light; origin of colors in thin film; Wedge shaped film; Newton's rings. Applications of interference - Determination of thickness of very thin wire or foil; determination of refractive index of liquid; wavelength of incident light; radius of curvature of lens; testing of surface flatness; Anti-reflecting films and Highly reflecting film.

Q. What is interference?

- Superposition of waves of light when two or more light waves are arriving at a point of the medium simultaneously is known as interference.
- Due to interference modification in the intensity of light will be takes place.
- The point at which light waves are interfering will appear bright (constructive interference) or dark (destructive interference) depends upon optical path difference between interfering light waves.



Q. What are the conditions for sustained or stationary interference fringes?

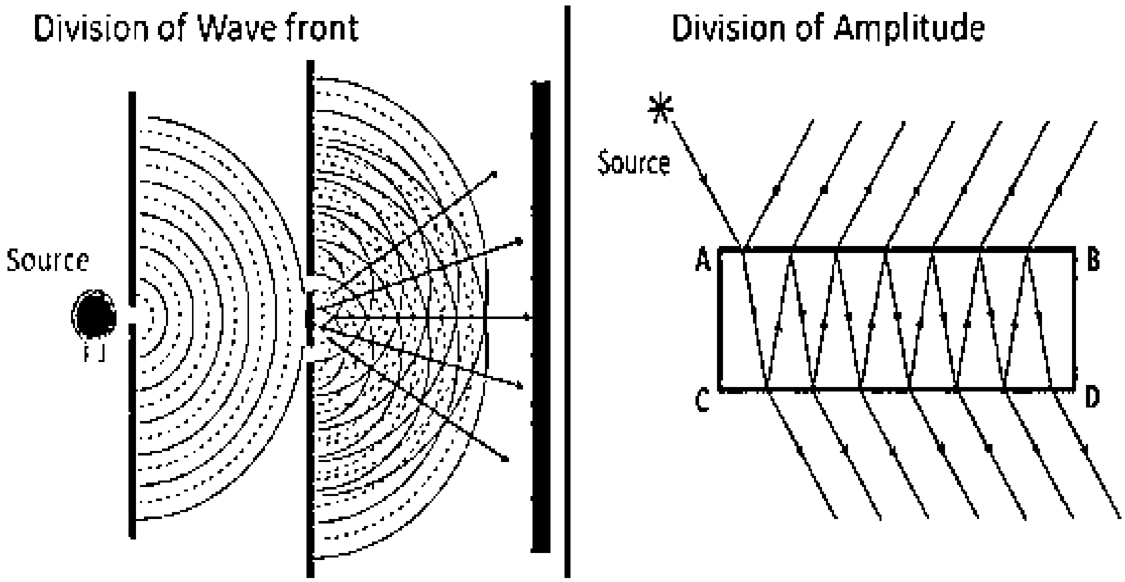
To obtain the sustained or stationary interference pattern light waves must be

- **Coherent: -** The phase difference between the waves remains constant. If the light waves are not coherent, the interference pattern will change continuously.
- **Monochromatic: -** The wavelengths of the light waves remain same so that interference

pattern will be sharp and well defined. If the light waves of different wavelengths, then interference fringes of different wavelengths are overlap on each other.

- **Same intensity: -** The light waves must have same intensity or amplitude to get good contrast of interference fringes.

There are two methods for obtaining light waves of same phase, wavelength and intensity



1) **Division of wave front:** The numbers of identical light rays are obtained by dividing the incident wave front of the light in to the number light rays using slits of constant width.

2) **Division of amplitude:** The incident light radiation is divided in to number of identical light rays of decreasing intensity using multiple reflections with in thin film of transparent material.

Q. What is the need of extended source of light to observe interference pattern of thin film?

1) When the thin film of transparent material is illuminated by white light, then condition of constructive interference is satisfied by different wavelengths at different angle of incidence and different colors are observed.
2) If the film is illuminated by point source of light, the incident light rays illuminate the film by different angle of incidence. The divergence of reflected light rays from the film is large and only small portion of the film i.e single color will be visible at a particular angle as the field of view is small. The observer will have to change his position to observe another color.

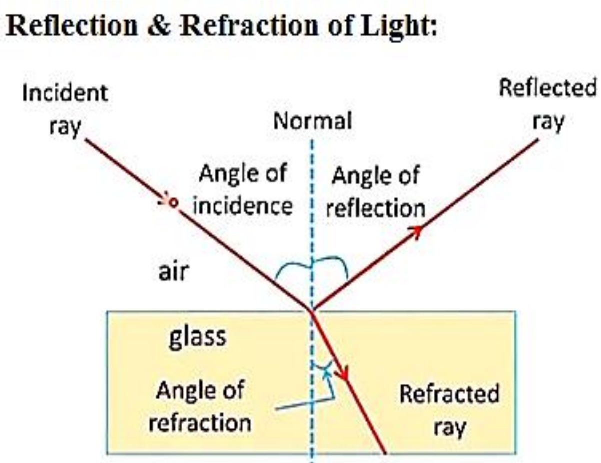
3) If the film is illuminated by extended i.e broad source of light, the incident light rays illuminate the film by different angle of incidence. The divergence of reflected light rays from the film is small and all portion of the film i.e all colors will be visible at a particular angle as the field of view is large.
4) If the film is illuminated by plane wave front of white light, the incident light rays are parallel and angle of incidence is same. Therefore film wills reflects only one color for which condition of constructive interference is satisfied at that particular angle of incidence.

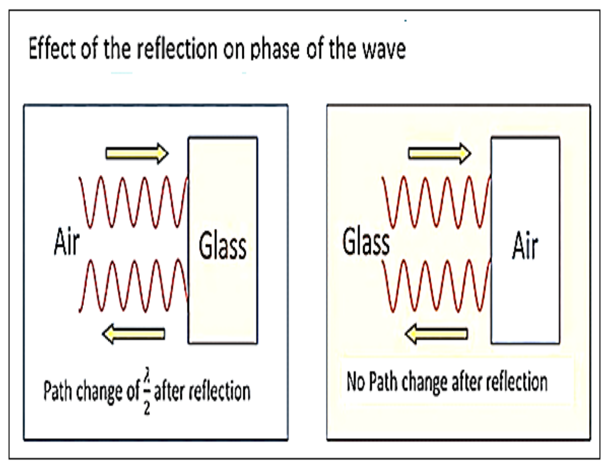
Q. What is thin film?

A film of transparent material of thickness approximately equal to average wavelength of the visible light (i.e 5500 AU) is known as thin film.

Q. What is Stokes law?

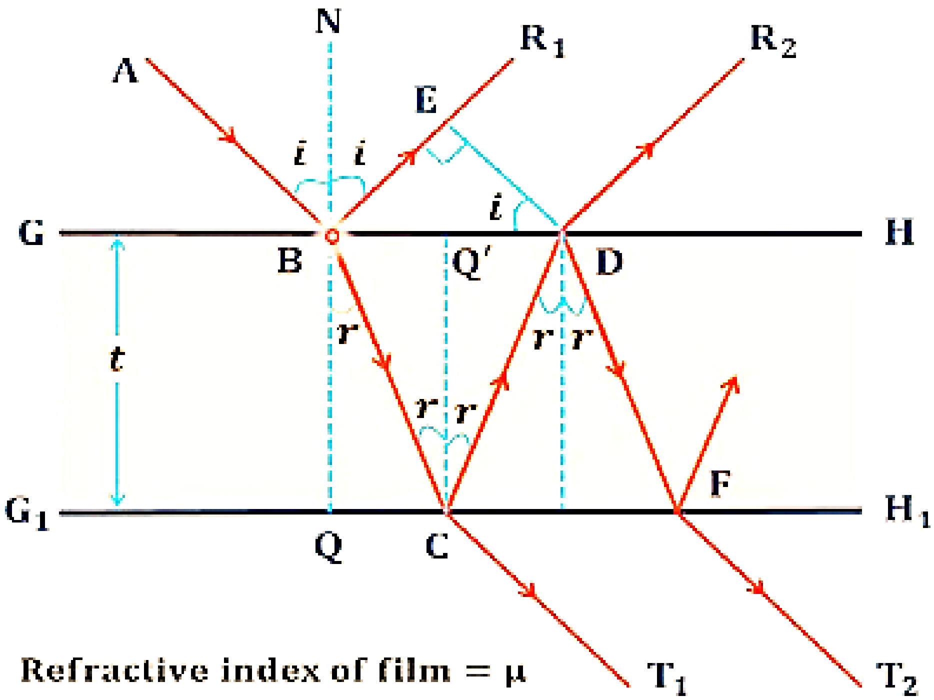
Whenever there is reflection of light from the optically denser medium, then phase change of π or path difference of λ/2 is produce between the reflected light rays.





Q. Derive the conditions of interference with in thin film of transparent material of uniform thickness? Why the fringe system is complementary in nature?

Interference with in a thin film of transparent material of uniform thickness:



1) Consider a thin film bounded between two surfaces S1 andS2.

   μ- R.I. of the film and

   t – Thickness of the film.

   The film is illuminated by the monochromatic source of light of wave length λ. depending upon the conditions of interference; the entire film is appeared dark or bright.

2) The incident light beam AB make an angle i with normal to the film. Due to multiple reflections with in a thin film, we get reflected light rays R1, R2…… and transmitted light rays T1, T2…..

3) To determine optical path difference between reflected light rays R1 and R2 draw perpendicular CQ’ and DE.

4) The optical path difference between reflected light rays R1 & R2 is

   Δ = Path (BC + CD) with in a film – Path BE in air

   Δ = (BC + CD) μ - BE------------ (1)   since μ = 1 for air.

From ΔBQC and $\Delta DQ'C$, we can write -

$$BC = CD = \frac{t}{\cos r} \qquad (2)$$

To determine BE, in Δ BED, sin i = BE/BD

$$BE = BD \sin i$$

$$= 2\, BQ' \sin i \qquad \text{As } \tan r = \frac{BQ'}{CQ'} = \frac{BQ'}{t}$$

$$= 2\, t \tan r \sin i$$

$$= 2\, t \frac{\sin r}{\cos r} \sin i$$

$$= 2\, t \frac{\sin r}{\cos r} \frac{\sin i}{\sin r} \sin r \qquad \text{As } \frac{\sin i}{\sin r} = \mu$$

$$\therefore BE = 2\, \mu\, t \frac{\sin^2 r}{\cos r} \qquad (3)$$

Eqn (1) =>

$$\Delta = \mu \left(\frac{t}{\cos r} + \frac{t}{\cos r}\right) - 2\, \mu\, t \frac{\sin^2 r}{\cos r}$$

$$\therefore \Delta = \frac{2\, \mu\, t}{\cos r}(1 - \sin^2 r)$$

$$\therefore \Delta = \frac{2\, \mu\, t}{\cos r} \cos^2 r$$

$$\therefore \Delta = 2\, \mu\, t \cos r \qquad (4)$$

As One reflection at point B occurs at the surface at denser medium. So additional path change of $\frac{\lambda}{2}$ is introduced. And effective optical path difference becomes -

$$\boxed{\therefore \Delta = 2\, \mu\, t \cos r \pm \frac{\lambda}{2}}$$

Condition for Maximum:	Condition for Minimum:
$\Delta = n\lambda$	$\Delta = (2n \pm 1)\frac{\lambda}{2}$
where $n = 0, 1, 2, 3 \ldots$	where $n = 0, 1, 2 \ldots$
$\therefore \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$	$\therefore \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$
$\therefore 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$	$\therefore 2\mu t \cos r = n\lambda$

For transmitted light, there is no phase change or path difference change between incident and transmitted light rays, therefore effective optical path difference between incident & transmitted light rays is

$$\boxed{\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}}$$

Therefore

Condition for Maximum:	Condition for Minimum:
$\Delta = n\lambda$	$\Delta = (2n \pm 1)\frac{\lambda}{2}$
where $n = 0, 1, 2, 3 \ldots$	where $n = 0, 1, 2 \ldots$
$\therefore \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$	$\therefore \Delta = 2\mu t \cos r \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$
$\therefore 2\mu t \cos r = (2n \pm 1)\frac{\lambda}{2}$	$\therefore 2\mu t \cos r = n\lambda$

Thus fringe system is complimentary in nature.

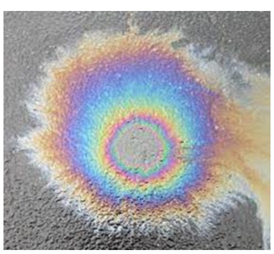
Formation of colors with in a thin film:

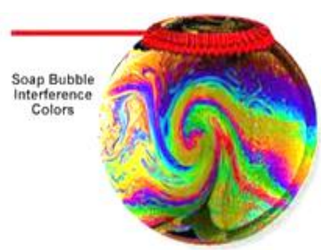
Q. Explain the formation of colors when thin film of transparent material is illuminated by sunlight or white light?

1) When thin film of transparent material (oil film, soap film, wings of butterfly) is illuminated by sunlight or white light different colors are observed on thefilm.
2) The variation in optical path difference will change the wavelength as well as intensity of reflected light rays and different color patches are observed o the film.
3) The O.P.D. between reflected light rays from the thin film is given by

$$\Delta = 2\mu t \cos r - \lambda/2$$

4) O.P.D , Δ varies with **μ, t andr.**
5) Refractive index of the film μ is a function of λ. The white light consists of colors of different wavelengths.
6) The angle of refraction varies with angle of incident i.e. angle of vision. Therefore variations of colors are observed on the thin film.

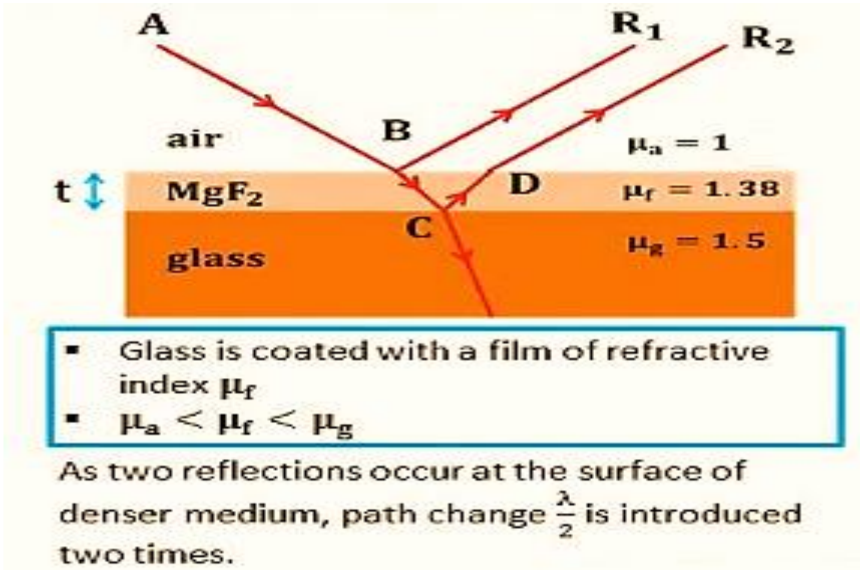




Nonreflecting lens or Anti reflection film:

Q. Write short notes on antireflection film? Give its applications?

Q. What is antireflection film? Derive amplitude & phase condition?



To avoid the reflection of light from the lens surface and to increase its transmittance, the surface of the lens in optical instrument like camera, telescope, binoculars, panel of solar cell etc. is coated with thin film of transparent material known as antireflection film. When light is incident on the glass surface, the intensity of reflected light is given by

$$Ir = \frac{\mu g-\mu a^2}{\mu g+\mu a} . Ii$$

Since μg = 1.5 and μa = 1

$$Ir = 0.04Ii$$

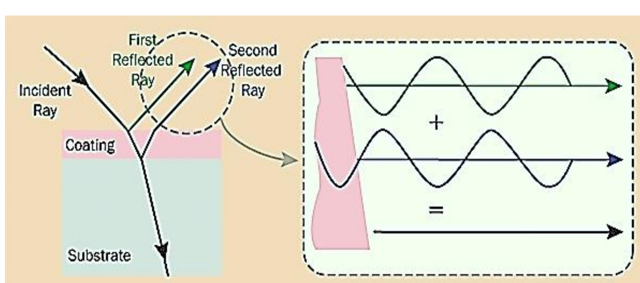
$$Ir = 4 \%Ii$$

Thus 4 % of incident light intensity is lost due to each reflection.

From figure, to avoid reflection, light rays R1 & R2 must interfere destructively.

There are two conditions

a) **Amplitude condition**:-



Amplitude of R1 = amplitude of R2

$$\left(\frac{\mu f - \mu a}{\mu f + \mu a}\right)^2 = \left(\frac{\mu g - \mu f}{\mu g + \mu f}\right)^2$$

On solving, we get $\mu f = \sqrt{\mu g}$

Thus R.I. of thin film material must be in between 1 & 1,5. The suitable material are MgF_2 (1.38) and Cryolite (1.36).

b) **Phase condition**:-

The phase or optical path difference between R1 & R2 must be λ/2.

$$\Delta = 2\mu_f t \cos r = \lambda/2$$

For normal incidence cos r = 1

Therefore, required thickness of the antireflection film is **t =** $\frac{\lambda}{4\mu_f}$

Highly reflecting film or lens:-

Q. Write short notes on highly reflection film? Give its applications?

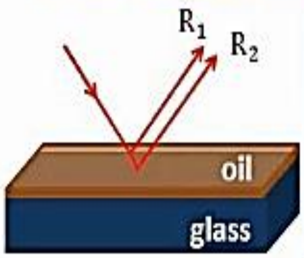
To avoid the transmission of light from the lens surface and to increase its reflectivity, the surface of the lens in optical instrument like sun glasses, windows pane etc. is coated with thin film of transparent material known as highly reflecting film. Such types of the lenses or glasses are used in summer to avoid heat in summer.

To increase the reflectivity of the lens surface, the reflected light rays R1 & R2 must interfere constructively. For constructive interference, the phase or optical path difference between R1 &R2 must be λ. $\Delta = 2\mu_f t \cos r = \lambda$

For normal incidence cos r = 1

Therefore, required thickness of the highly reflecting film is **t = λ / 2μf**

A plane wave of monochromatic light falls normally on a uniformly thin film of oil having refractive index 1.3 which covers a glass plate of refractive index 1.5. The wavelength of the source can be varied continuously. Complete destructive interference is obtained for wavelength 5000 A⁰ and 7000 A⁰ and no other wavelength in between. Find the thickness of the oil layer.



Given : $\mu_{oil} = 1.3$

$\mu_{glass} = 1.5$

For normal incidence, $r = 0$

$\lambda_1 = 7000A^0 = 7000 \times 10^{-8}$ cm

$\lambda_2 = 5000A^0 = 5000 \times 10^{-8}$ cm

thickness = ?

Effective optical path difference between two reflected rays in reflected system is -

$$\Delta = 2\,\mu_{oil}\,t\cos r \pm \frac{\lambda}{2} \pm \frac{\lambda}{2} = 2\,\mu_{oil}\,t\cos r$$

For $(n_1)^{th}$ order destructive interference of λ_1

$$2\,\mu_{oil}\,t\cos r = (2n_1 - 1)\frac{\lambda_1}{2} \quad (1)$$

For $(n_2)^{th}$ order destructive interference of λ_2

$$2\,\mu_{oil}\,t\cos r = (2n_2 - 1)\frac{\lambda_2}{2} \quad (2)$$

Divide (1) by (2)

$$\therefore \frac{2n_1 - 1}{2n_2 - 1} = \frac{\lambda_2}{\lambda_1} = \frac{5000}{7000} = \frac{5}{7}$$

$$\therefore 2n_1 - 1 = 5 \Rightarrow n_1 = 3 \text{ and } 2n_2 - 1 = 7 \Rightarrow n_2 = 4$$

Putting $n_1 = 3$ and $\lambda_1 = 7000A^0 = 7000 \times 10^{-8}$ cm in (1) thickness (t) can be found

$$t = \frac{(2n_1 - 1)\frac{\lambda_1}{2}}{2\,\mu_{oil}\cos r} = \frac{(2(3) - 1) \times \frac{7000 \times 10^{-8}}{2}}{2 \times 1.3 \times \cos 0}$$

$$= 6.73 \times 10^{-5} \text{ cm} = 6730\ A^0$$

White light is incident on a soap film (μ = 1.33) at an angle $\sin^{-1}(4/5)$ and the reflected light on examination shows dark bands. Two consecutive dark bands corresponding to wavelengths 6100 A⁰ and 6000 A⁰ coincide with each other. Calculate the thickness of the film.

Given : $\mu = 1.33$, $i = \sin^{-1}(4/5)$, $\therefore \sin i = 4/5$

$\lambda_1 = 6100A^0 = 6100 \times 10^{-8}$ cm, $\lambda_2 = 6000A^0, = 6000 \times 10^{-8}$ cm thickness = ?

$$\cos r = \sqrt{1 - \sin^2 r} = \sqrt{1 - \frac{\sin^2 i}{\mu^2}} = \sqrt{1 - \frac{(4/5)^2}{(1.33)^2}} = 0.7988$$

For minimum (dark band) in reflected light,

$$2\,\mu\,t\cos r = n\,\lambda$$

$\therefore$ For n^{th} dark band of λ_1 in reflected light,

$$2\,\mu\,t\cos r = n\,\lambda_1 \quad (1)$$

$\therefore$ For $(n+1)^{th}$ dark band of λ_2 in reflected light,

$$2\,\mu\,t\cos r = (n+1)\lambda_2 \quad (2)$$

As n^{th} dark band of λ_1 and $(n+1)^{th}$ dark band of λ_2 coincide

$$\therefore n\,\lambda_1 = (n+1)\lambda_2$$

$$\therefore n\,\lambda_1 = n\lambda_2 + \lambda_2$$

$$\therefore n = \frac{\lambda_2}{\lambda_1 - \lambda_2} = \frac{6000}{6100 - 6000} = 60$$

Putting n=60 in (1) thickness (t) can be found

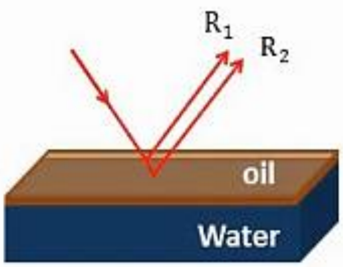
$$t = \frac{n\,\lambda_1}{2\,\mu\cos r} = \frac{60 \times 6100 \times 10^{-8}}{2 \times 1.33 \times 0.7988}$$

$$= 1.72 \times 10^{-3} \text{ cm}$$

A drop of oil of volume 0.2 cc is dropped on the surface of a tank water of area 1sq.m. The oil film spreads uniformly over the whole surface. White light which is incident normally on the surface is observed through spectroscope. The spectrum is seen to contain one dark band whose centre has wavelength 5500 A^0 in air. Find the refractive index of the given oil.

Given : For normal incidence, $r = 0$, Volume $= 0.2\ cm^3$,

Area $= 1m^2 = 10^4 cm^2$, $\lambda = 5500\ A^0 = 5500 \times 10^{-8}$ cm, $\mu = ?$



$$\text{Thickness of the oil film} = \frac{\text{Volume}}{\text{Area}} = \frac{0.2}{10^{-4}} = 2 \times 10^{-5}\ \text{cm}$$

Dark band means we have to apply condition for Minimum in reflected system

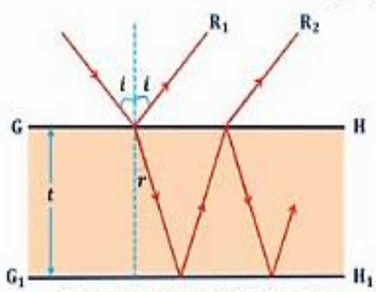
i.e. $2\mu t \cos r = n\lambda$ Let $n = 1$

$$\therefore \mu = \frac{n\lambda}{2t\cos r} = \frac{1 \times 5500 \times 10^{-8}}{2 \times 2 \times 10^{-5} \times \cos 0} = 1.375$$

A glass plate having thickness of 0.4 x 10^{-4} cm is illuminated by a beam of white light normal to the plate (μ for glass = 1.5). Calculate the wavelength within the limits of visible spectrum (λ = 4000 A^0 to 7000 A^0) which will be intensified in the reflected beam.

Given : $t = 0.4 \times 10^{-4}$ cm, $\mu = 1.5$, For normal incidence, $r = 0$

Intensified means we have to apply condition for Maximum in reflected system



Refractive index of film = μ

For Maximum in reflected system, $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$

$$\therefore \lambda = \frac{2 \times 2\mu t \cos r}{(2n-1)} \qquad n = 1,2,3,\ldots$$

$$\therefore \text{For } n = 1, \quad \lambda = \frac{2 \times 2 \times 1.5 \times 0.4 \times 10^{-4} \times \cos 0}{(2 \times 1 - 1)} = 24000\ A^0$$

$$\therefore \text{For } n = 2, \quad \lambda = \frac{2 \times 2 \times 1.5 \times 0.4 \times 10^{-4} \times \cos 0}{(2 \times 2 - 1)} = 8000\ A^0$$

$$\therefore \text{For } n = 3, \quad \lambda = \frac{2 \times 2 \times 1.5 \times 0.4 \times 10^{-4} \times \cos 0}{(2 \times 3 - 1)} = 4800\ A^0$$

$$\therefore \text{For } n = 4, \quad \lambda = \frac{2 \times 2 \times 1.5 \times 0.4 \times 10^{-4} \times \cos 0}{(2 \times 4 - 1)} = 3428\ A^0$$

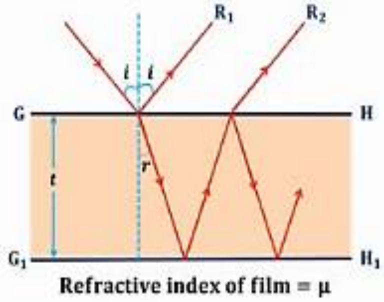
Effective path difference between ray R_1 and R_2 is

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

White light falls normally on a soap film of refractive index 1.33 and thickness 5000 A⁰. What wavelength within the visible spectrum (λ = 4000 A⁰ to 7000 A⁰) will be strongly reflected?

Given : $\mu = 1.33$, $t = 5000\ A^0 = 5000 \times 10^{-8}$ cm, For normal incidence, $r = 0$

Strongly reflected means we have to apply condition for Maximum in reflected system



Effective path difference between ray R_1 and R_2 is

$$\Delta = 2\mu t \cos r \pm \frac{\lambda}{2}$$

For Maximum in reflected system, $2\mu t \cos r = (2n-1)\frac{\lambda}{2}$

$$\therefore \lambda = \frac{2 \times 2\mu t \cos r}{(2n-1)} \qquad n = 1,2,3,\ldots$$

$$\therefore \text{For } n = 1, \quad \lambda = \frac{2 \times 2 \times 1.33 \times 5000 \times \cos 0}{(2 \times 1 - 1)} = 26600\ A^0$$

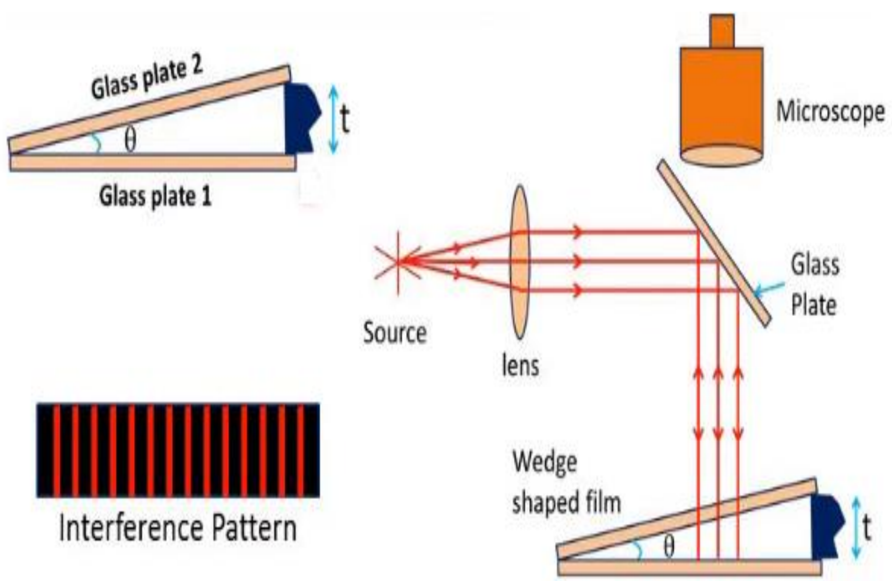
$$\therefore \text{For } n = 2, \quad \lambda = \frac{2 \times 2 \times 1.33 \times 5000 \times \cos 0}{(2 \times 2 - 1)} = 8867\ A^0$$

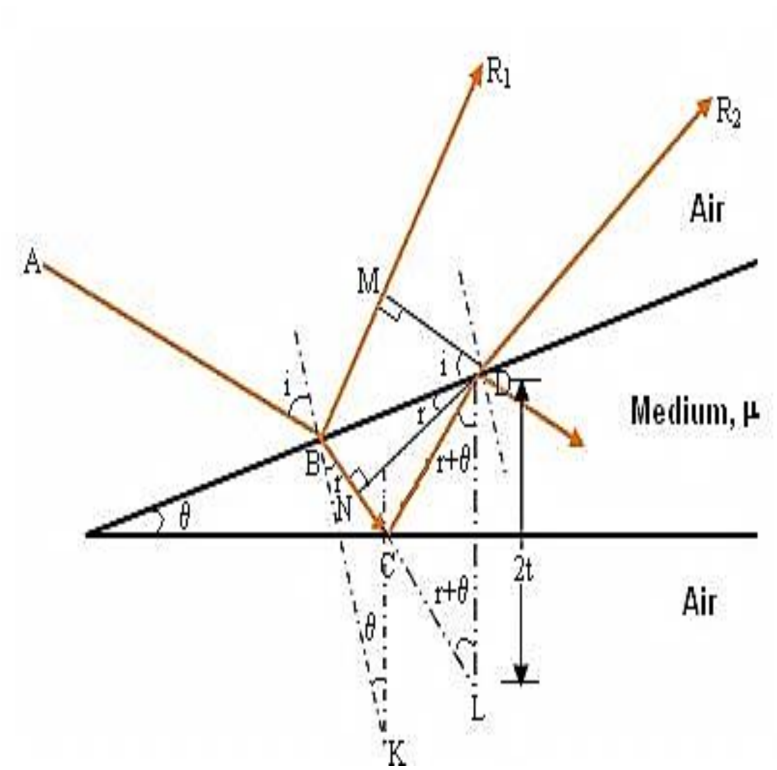
$$\therefore \text{For } n = 3, \quad \lambda = \frac{2 \times 2 \times 1.33 \times 5000 \times \cos 0}{(2 \times 3 - 1)} = 5320\ A^0$$

$$\therefore \text{For } n = 4, \quad \lambda = \frac{2 \times 2 \times 1.33 \times 5000 \times \cos 0}{(2 \times 4 - 1)} = 3800\ A^0$$

Interference with in a thin film of wedge shape:

Q. Derive the conditions of interference with in thin film of transparent material of wedge shape?





Due to interference with in a thin film of wedge shape film, alternate dark and bright bands of constant thickness parallel to surface of the upper glass plate are observed. These fringes of constant thickness are known as Fizeau fringes.

The optical path difference between reflected light rays R1 & R2 is

Δ = Path (BC + CD) with in a film – Path BM in air

Δ = (BC + CD) μ - BM   Since μ = 1 for air.

$\Delta = 2\mu t \cos(r + Q) - \lambda/2$

For constructive interference (bright band) in reflected light

$\Delta = n\lambda$

$2\mu t \cos(r + Q) = (2n + 1)\lambda/2$

For destructive interference (dark band) in reflected light

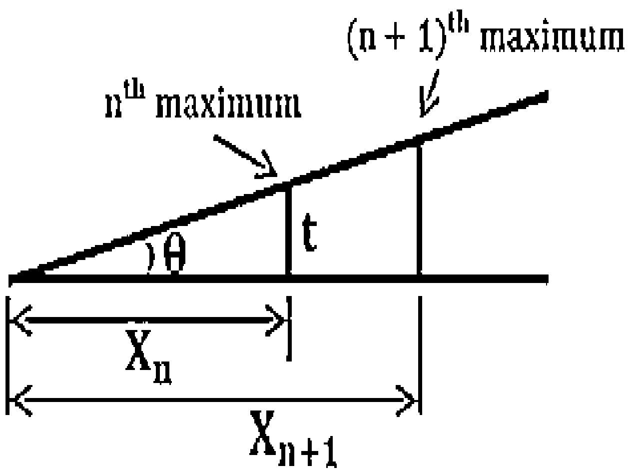
$\Delta = (2n + 1)\lambda/2$

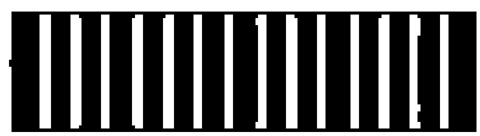
$2\mu t \cos(r + Q) = n\lambda$

Fringe width:-

Q. What is fringe width in wedge shape film interference? Derive its expression?

1) **The distance between two consecutive dark or bright bands in the interference fringes of wedge shaped film is known as fringe width.**





2) Consider two glass plate P1 & P2 inclined with small angle α. An air film of wedge shape is formed between the glass plates with angle of wedge ϴ. The film is illuminated by the monochromatic source of light of wave length λ.
3) Due to interference with in a thin film of wedge shape film, alternate dark and bright bands of constant thickness parallel to surface of the upper glass plate are observed. These fringes of constant thickness are known as Fizeau fringes.
4) Let n^{th} dark band is at x_n and $(n+1)^{th}$ dark band is at x_{n+1} from the apex of the film where thickness of the film is t_n and t_{n+1} respectively.

$x_{n+1} - x_n = \beta$ is known as fringe width

For n^{th} maximum, we have -

$$2\,\mu\, t \cos(r+\theta) = (2n-1)\frac{\lambda}{2} \qquad n = 1,2,3\ldots$$

For normal incidence, $r = 0$

$$\therefore 2\,\mu\, t \cos\theta = (2n-1)\frac{\lambda}{2} \quad (1)$$

Let n^{th} maximum is obtained at distance X_n from the edge

From figure, $\tan\theta = \frac{t}{X_n} \Rightarrow t = X_n \tan\theta$

(1) becomes

$$\therefore 2\mu X_n \tan\theta \cos\theta = (2n-1)\frac{\lambda}{2}$$

$$\therefore 2\mu X_n \sin\theta = (2n-1)\frac{\lambda}{2} \quad (2)$$

Similarly, for $(n+1)^{th}$ maximum,

$$\therefore 2\mu X_{n+1} \sin\theta = (2(n+1)-1)\frac{\lambda}{2} \quad (3)$$

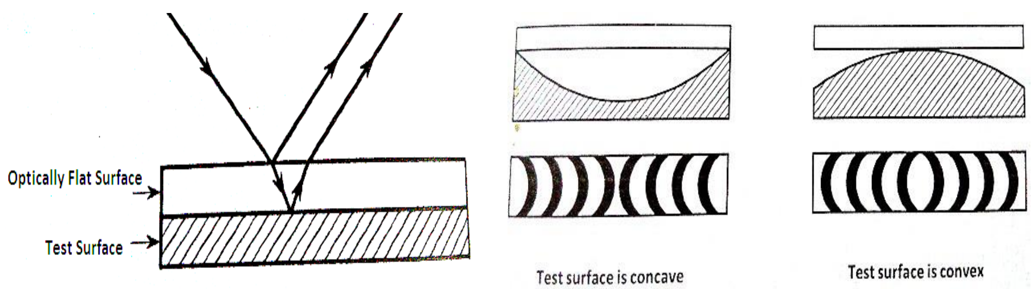
Subtract (2) from (3)

$$X_{n+1} - X_n = \frac{\lambda}{2\mu\sin\theta} = \text{fringewidth } (\beta)$$

Application of wedge shaped film interference:

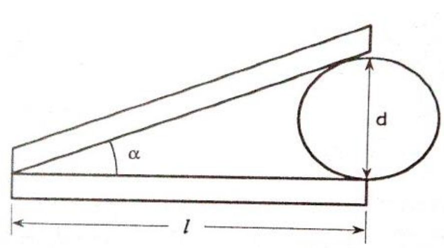
1) To test optical flatness of glass plate:

Q. Write short note on optical flatness?

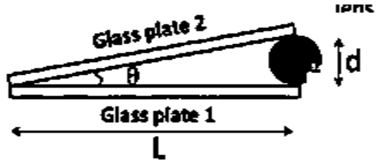


In optical instrument, perfectly flat surface of the glass plate or smooth curved surface of the lens is required. The phenomenon of interference of light with in thin film is used to test the flat surface or curved surface of the glass plate. The glass plate or the lens under the test is kept on the reference glass plate which having perfectly flat surface to form wedge shaped thin film. The thin film is illuminated by monochromatic light and fringes of constant thickness are observed. The glass plate under the test is polished till we get the fringes of constant thickness.

2) To determine diameter of thin wire or thickness of thin metal foil:-



A wire whose diameter is to be measured is kept at a distance 'l' between two glass plate to form wedge shaped thin film. The thin film is illuminated by monochromatic light of known wavelength λ. The fringes of constant thickness are observed. The fringe width is measured using microscope.



$$\text{fringewidth } (\beta) = \frac{\lambda}{2\,\mu \sin\theta}$$

$$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{2\,\mu\,\beta}\right)$$

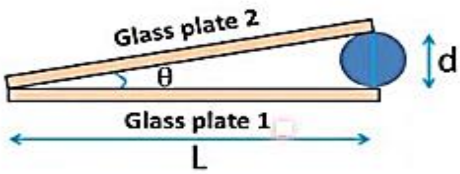
$$\therefore \text{diameter } (d) = L \tan\theta$$

An air wedge is formed by keeping a fine wire at one edge between two glass plates. When the film is illuminated normally with light of wavelength 550 nm, fringe-width of the fringes observed is 1 mm. Calculate the diameter of the wire if the length of the plate is 5cm.

Given : $\lambda = 550\text{nm} = 550 \times 10^{-9}\text{m}$

fringewidth $\beta = 1 \text{ mm} = 10^{-3} \text{ m}$

Length of the plate $L = 5 \text{ cm} = 5 \times 10^{-2}\text{m}$

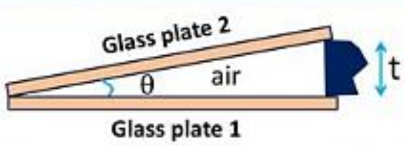


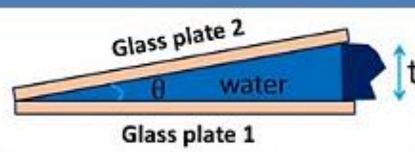
$$\text{fringewidth } (\beta) = \frac{\lambda}{2\,\mu \sin\theta}$$

$$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{2\,\mu\,\beta}\right) = \sin^{-1}\left(\frac{550\times10^{-9}}{2\times1\times10^{-3}}\right) = 0.0157^{0}$$

$$\therefore \text{diameter } (d) = L \tan\theta = 5\times10^{-2}\times\tan(0.0157) = 1.37\times10^{-5}\text{m}$$

When a wedge shaped air film is viewed by a monochromatic source of light incident normally, the interference fringes 0.4 mm apart are observed. If the air space is filled with water (μ = 1.33), how far apart will the fringes be observed?





$$\text{fringewidth } (\beta_{water}) = \frac{\lambda}{2\,\mu \sin\theta} \quad (1)$$

$$\text{fringewidth } (\beta_{air}) = \frac{\lambda}{2\sin\theta} \quad (2)$$

Given : $\beta_{air} = 0.4 \text{ mm} = 0.04 \text{ cm}$

Refractive index of water 1.33

Interference Pattern

Divide (2) by (1)

$$\frac{\beta_{air}}{\beta_{water}} = \frac{\frac{\lambda}{2\sin\theta}}{\frac{\lambda}{2\,\mu\sin\theta}} = \mu$$

$$\therefore \beta_{water} = \frac{\beta_{air}}{\mu} = \frac{0.04}{1.33} = 0.03 \text{ cm}$$

Fringes of equal thickness are observed in a thin glass wedge of refractive index 1.52. The fringe spacing obtained with light of wavelength 5893A⁰ is 0.1 mm. Calculate angle of wedge.

Given : $\lambda = 5893\ A^0 = 5893 \times 10^{-8}$ cm,

Fringe width $\beta = 0.1$ mm $= 0.01$ cm

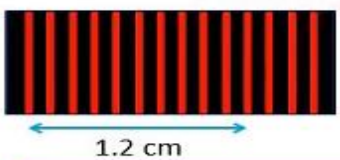
Refractive index $\mu = 1.52$

$$\text{fringewidth } (\beta) = \frac{\lambda}{2\,\mu \sin\theta}$$

$$\therefore \theta = \sin^{-1}\left(\frac{\lambda}{2\,\mu\,\beta}\right) = \sin^{-1}\left(\frac{5893 \times 10^{-8}}{2 \times 1.52 \times 0.01}\right) = 0.111^0$$

A wedge shaped air film having angle of 40 seconds is illuminated by monochromatic light. Fringes are observed vertically through a microscope. The distance between ten consecutive dark fringes is 1.2 cm. Find the wavelength of monochromatic light.

Given : $\theta = 40$ seconds $= \frac{40}{3600}$ degrees, For air film, $\mu = 1$



$$\text{fringewidth } (\beta) = \frac{1.2}{10} = 0.12 \text{ cm}$$

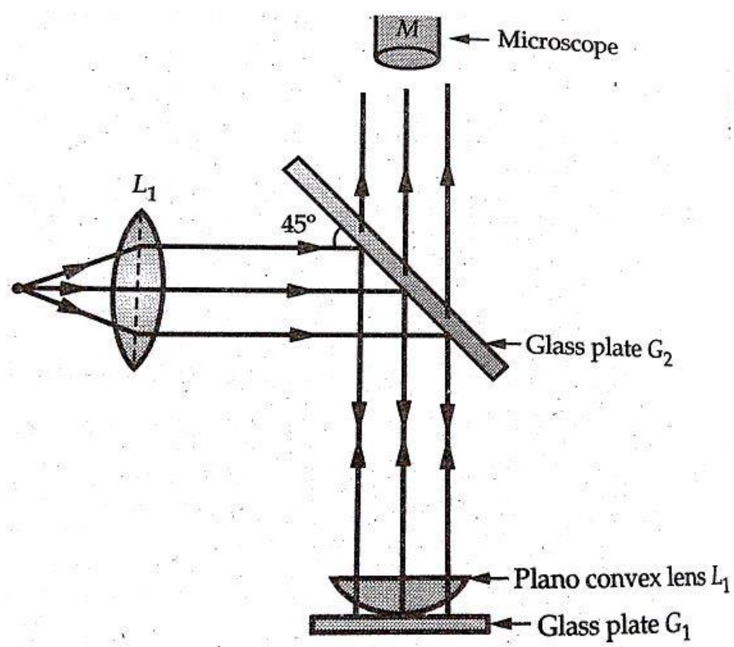
$$\text{fringewidth } (\beta) = \frac{\lambda}{2\,\mu \sin\theta}$$

$$\therefore \lambda = 2\,\mu \sin\theta \times \beta = 2 \times 1 \times \sin\left(\frac{40}{3600}\right) \times 0.12$$

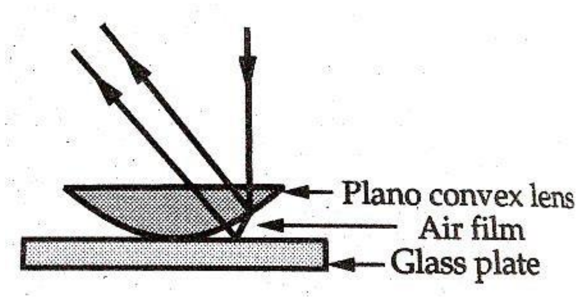
$$= 4.654 \times 10^{-5} \text{cm} = 4654\ A^0$$

Interference with in thin film of varying thickness (Newton's rings):-

Q, Explain Newton's rings experiment?



When Plano convex lens of large radius of curvature is kept on a plane glass p[late, then an air film of varying thickness is enclosed between convex surface of the lens and top surface of the glass plate. The thickness of the film is zero at the point of contact of the lens and the glass plate and thickness increases away from the point of contact along the circumference of the lens. When such a film of varying thickness is illuminated by monochromatic light, then number of dark and bright rings concentric around the point of contact of the lens and the glass plate are observed due to interference of light with in a thin of varying thickness. These rings are known as Newton's rings.

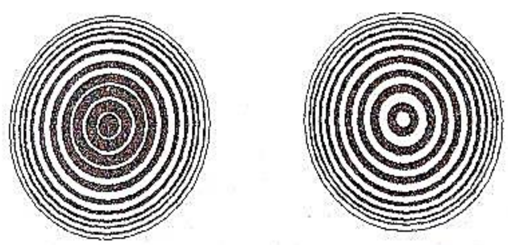


For constructive interference (bright rings) in reflected light

$2\mu t \cos(r + \alpha) = (2n + 1)\lambda/2$

For destructive interference (dark rings) in reflected light

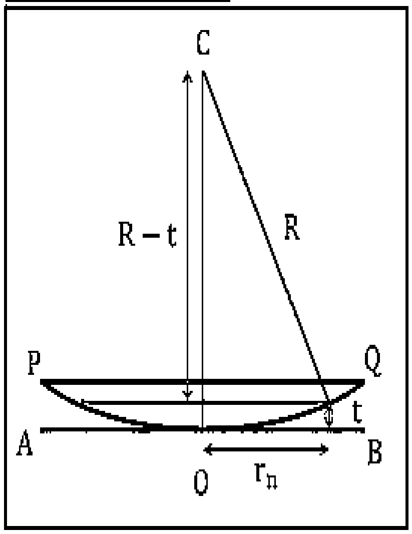
$2\mu t \cos(r + \alpha) = n\lambda$



(a) Newton's ring in reflected system (b) Newton's ring in transmitted syste

Radius or diameter of Newton’s ring:-

Q. Show that radius or diameter of Newton’s dark rings is directly proportional to square root of natural number?



Consider the lens and glass plate to form thin film of varying thickness.

If R - Radius of curvature of the lens,

r_n - Radius of n^{th} dark ring,

t_n - Thickness of the film where n^{th} dark is obtained.

The optical path difference between two rays reflected from upper and lower surface of the film is given by –

$$\Delta = 2\,\mu\,t\cos(r+\theta) \pm \frac{\lambda}{2}$$

For normal incidence, r = 0

If R is very large, θ = 0

$$\therefore \Delta = 2\,\mu\,t \pm \frac{\lambda}{2} \quad (1)$$

Let n^{th} ring in the interference pattern is obtained at distance r_n from the point of contact.

$\therefore r_n$ is the radius of n^{th} ring

From the right angled triangle formed in figure,

$$R^2 = (R - t)^2 + (r_n)^2$$

$$\therefore R^2 = R^2 - 2Rt + t^2 + (r_n)^2$$

$$\therefore (r_n)^2 = 2Rt - t^2$$

As t is small, t^2 will be further small and can ne neglected

$$\therefore (r_n)^2 = 2Rt$$

$$\therefore 2t = \frac{(r_n)^2}{R} \quad (2)$$

$$\therefore \Delta = \frac{(r_n)^2}{R}\mu \pm \frac{\lambda}{2} \quad (3)$$

To get radius of n^{th} **dark ring**, apply condition for minimum

$$\Delta = (2n \pm 1)\frac{\lambda}{2} \qquad n = 0,1,2,3$$

$$\therefore \frac{(r_n)^2}{R}\mu \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\therefore \frac{(r_n)^2}{R}\mu = n\lambda$$

Radius of n^{th} **dark ring** is -

$$r_n = \sqrt{\frac{nR\lambda}{\mu}} \quad (4)$$

Diameter of n^{th} **dark ring** is -

$$D_n = \sqrt{\frac{4nR\lambda}{\mu}} \quad (5)$$

For air film, $\mu = 1$

$$\therefore D_n = \sqrt{4nR\lambda}$$

$\therefore$diameter of n^{th} **dark ring** is directly proportional to $\sqrt{n}$

Thus radius or diameter of nth dark ring is directly proportional to square root of natural number.

To get radius of n^{th} **bright ring**, apply condition for maximum

$$\Delta = n\lambda \qquad n = 0,1,2,3..$$

$$\therefore \frac{(r_n)^2}{R}\mu \pm \frac{\lambda}{2} = n\lambda$$

$$\therefore \frac{(r_n)^2}{R}\mu = (2n \pm 1)\frac{\lambda}{2}$$

∴Radius of n^{th} bright ring is -

$$r_n = \sqrt{\frac{(2n \pm 1)R\lambda}{2\,\mu}} \qquad (6)$$

∴Diameter of n^{th} bright ring is -

$$D_n = \sqrt{\frac{2(2n \pm 1)R\lambda}{\mu}} \qquad (7)$$

For air film, $\mu = 1$

$$\therefore D_n = \sqrt{2(2n \pm 1)R\lambda}$$

∴diameter of n^{th} **bright ring** is directly proportional to $\sqrt{(2n \pm 1)}$

Thus radius or diameter of nth bright ring is directly proportional to square root of odd the natural number.

Applications of Newton's ring's Experiment:

***Q*. Explain how Newton's ring experiment is used to determine radius of curvature or Wavelength of light source?**

1) **To determine radius of curvature or wavelength of light source-**

The Newton's rings are obtained using given lens and light source. The diameter of **n^{th}** and **$(m+n)^{th}$** dark rings are measured using microscope.

Diameter of n^{th} dark ring is given by-

$$D_n = \sqrt{4nR\lambda} \qquad \therefore (D_n)^2 = 4nR\lambda \quad (1)$$

Diameter of $(m+n)^{th}$ dark ring is given by-

$$\therefore (D_{m+n})^2 = 4(m+n)R\lambda \quad (2)$$

From (1) and (2)

$$R = \frac{(D_{m+n})^2 - (D_n)^2}{4m\lambda}$$

$$\lambda = \frac{(D_{m+n})^2 - (D_n)^2}{4mR}$$

These formulae are used to determine the radius of curvature or wave length of light source.

Q. Explain how Newton's ring experiment is used to determine the refractive index of any liquid or oil?

2) To determine refractive index of given liquid or oil:

The Newtons ring are obtained using air film & then liquid film between lens and glass plate.

The diameter of n^{th} dark rings are measured using microscope.

Diameter of n^{th} dark ring is

$$(D_n)_{liquid} = \sqrt{\frac{4nR\lambda}{\mu}} \quad (1)$$

For air film (refractive index = 1)

$$\therefore (D_n)_{air} = \sqrt{4nR\lambda} \quad (2)$$

Divide (2) by (1)

$$\frac{(D_n)_{air}}{(D_n)_{liquid}} = \sqrt{\mu}$$

$$\mu = \left(\frac{(D_n)_{air}}{(D_n)_{liquid}}\right)^2$$

This formula is used to determine RI of liquid or oil.

Newton's rings are formed with reflected light of wavelength 5900 A⁰. The diameter of the third bright ring is 2 mm. If the space between the lens and the plate is filled with a liquid of refractive index = 1.33, calculate the radius of curvature of the lens.

Given: $\lambda = 5900\ A^0 = 5900 \times 10^{-8}$ cm, $n = 3$, $(D_3)_{liquid} = 2$ mm $= 0.2$ cm

Refractive index of liquid, $\mu = 1.33$ $R = ?$

Diameter if n^{th} bright ring is given by -,

$$D_n = \sqrt{\frac{2(2n-1)R\lambda}{\mu}}$$

$$\therefore R = \frac{(D_n)^2 \times \mu}{2(2n-1)\lambda}$$

$$\therefore R = \frac{(0.2)^2 \times 1.33}{2(2 \times 3 - 1) \times 5900 \times 10^{-8}}$$

$$= 90.17 \text{ cm}$$

A Newton's ring setup is used with a source emitting two wavelengths $\lambda_1 = 6000A^0$ and $\lambda_2 = 4500\ A^0$. , The n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to $\lambda 2$. If the radius of curvature of the lens is 90 cm, find the diameter of the n^{th} dark ring of 6000 A⁰.

Given: $\lambda_1 = 6000A^0 = 6000 \times 10^{-8}$ cm, $\lambda_2 = 4500\ A^0 = 4500 \times 10^{-8}$ cm

$R = 90$ cm For air film, $\mu = 1$ $D_n = ?$

Radius of n^{th} dark ring of λ_1 is given by -

$$(r_n)_{\lambda_1} = \sqrt{nR\lambda_1} \quad (1)$$

For $(n+1)^{th}$ dark ring of λ_2

$$(r_{n+1})_{\lambda_2} = \sqrt{(n+1)R\lambda_2} \quad (2)$$

n^{th} dark ring of λ_1 and

$(n+1)^{th}$ dark ring of λ_2 coincide

$$\therefore (r_n)_{\lambda_1} = (r_{n+1})_{\lambda_2}$$

$$\therefore nR\lambda_1 = (n+1)R\lambda_2$$

$$\therefore n\,\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore n\,(\lambda_1 - \lambda_2) = \lambda_2$$

$$\therefore n = \frac{4500}{(6000-4500)} = 3$$

Diameter due to nth dark ring of $\lambda_1 = 6000A^0$ is

$$(D_n)_{\lambda_1} = 2(r_n)_{\lambda_1} = 2\sqrt{nR\lambda_1}$$

$$= 2\sqrt{3 \times 90 \times 6000 \times 10^{-8}} = 0.2545 \text{ cm}$$

A Newton's ring setup is used with a source emitting two wavelengths $\lambda_1 = 6000A^0$ and $\lambda_2 = 4500\ A^0$. , The n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to $\lambda 2$. If the radius of curvature of the lens is 90 cm, find the diameter of the n^{th} dark ring of 6000 A⁰.

Given: $\lambda_1 = 6000A^0 = 6000 \times 10^{-8}$ cm, $\lambda_2 = 4500\ A^0 = 4500 \times 10^{-8}$ cm

$R = 90$ cm For air film, $\mu = 1$ $D_n = ?$

Radius of n^{th} dark ring of λ_1 is given by -

$$(r_n)_{\lambda_1} = \sqrt{nR\lambda_1} \quad (1)$$

For $(n+1)^{th}$ dark ring of λ_2

$$(r_{n+1})_{\lambda_2} = \sqrt{(n+1)R\lambda_2} \quad (2)$$

n^{th} dark ring of λ_1 and

$(n+1)^{th}$ dark ring of λ_2 coincide

$$\therefore (r_n)_{\lambda_1} = (r_{n+1})_{\lambda_2}$$

$$\therefore nR\lambda_1 = (n+1)R\lambda_2$$

$$\therefore n\,\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore n\,(\lambda_1 - \lambda_2) = \lambda_2$$

$$\therefore n = \frac{4500}{(6000-4500)} = 3$$

Diameter due to nth dark ring of $\lambda_1 = 6000A^0$ is

$$(D_n)_{\lambda_1} = 2(r_n)_{\lambda_1} = 2\sqrt{nR\lambda_1}$$

$$= 2\sqrt{3 \times 90 \times 6000 \times 10^{-8}} = 0.2545 \text{ cm}$$

Light containing two wavelengths λ_1 and λ_2 falls normally on a convex lens of radius of curvature R, resting on a glass plate. Now, if n^{th} dark ring due to λ_1 coincides with $(n+1)^{th}$ dark ring due to λ_2, then prove that the radius of the n^{th} dark ring due to λ_1 is $\sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1-\lambda_2}}$

Radius of n^{th} dark ring of λ_1 is given by -

$$(r_n)_{\lambda_1} = \sqrt{\frac{nR\lambda_1}{\mu}} \quad (1)$$

For $(n+1)^{th}$ dark ring of λ_2

$$(r_{n+1})_{\lambda_2} = \sqrt{\frac{(n+1)R\lambda_2}{\mu}} \quad (2)$$

n^{th} dark ring of λ_1 and
$(n+1)^{th}$ dark ring of λ_2 coincide

$$\therefore (r_n)_{\lambda_1} = (r_{n+1})_{\lambda_2}$$

$$\therefore \frac{nR\lambda_1}{\mu} = \frac{(n+1)R\lambda_2}{\mu}$$

$$\therefore n\,\lambda_1 = (n+1)\,\lambda_2$$

$$\therefore n\,(\lambda_1-\lambda_2) = \lambda_2 \qquad \therefore n = \frac{\lambda_2}{(\lambda_1-\lambda_2)}$$

Putting in (1)

$$(r_n)_{\lambda_1} = \sqrt{\frac{\lambda_2}{(\lambda_1-\lambda_2)}\frac{R\lambda_1}{\mu}} = \sqrt{\frac{\lambda_1\lambda_2 R}{\lambda_1-\lambda_2}}$$

In Newton's ring experiment, the diameter of the 4^{th} and 12^{th} dark rings are 0.4 cm and 0.7 cm respectively. Find the diameter of the 20^{th} dark ring.

Given: $D_4 = 0.4$ cm, $D_{12} = 0.7$ cm, Assume air film $D_{20} = ?$

Diameter of n^{th} dark ring is given by

$$\therefore (D_n)^2 = 4nR\lambda \qquad \text{For air film, } \mu = 1$$

$$\therefore (D_4)^2 = 4 \times 4\ R\lambda$$

$$\therefore 0.4^2 = 4 \times 4\ R\lambda$$

$$\therefore 0.16 = 16\ R\lambda \quad (1)$$

$$\therefore (D_{12})^2 = 4 \times 12\ R\lambda$$

$$\therefore 0.7^2 = 4 \times 12\ R\lambda$$

$$\therefore 0.49 = 48\ R\lambda \quad (2)$$

Subtract (1) from (2)

$$48\ R\lambda - 16\ R\lambda = 0.49 - 0.16$$

$$\therefore 32\ R\lambda = 0.33$$

$$\therefore R\lambda = \frac{0.33}{32}$$

$$\text{Now, } (D_{20})^2 = 4 \times 20\ R\lambda$$

$$\therefore D_{20} = \sqrt{4 \times 20\ R\lambda} = \sqrt{4 \times 20 \times \frac{0.33}{32}}$$

$$= 0.908 \text{ cm}$$

The diameter of 10^{th} dark ring is 5 mm, when light of wavelength 5500 A^0 is used in Newton's rings experiment. If the space between lens and glass plate is filled with a liquid of refractive index 1.25, what will be the diameter of 10^{th} dark ring?

Given: $\lambda = 5500\ A^0 = 5500 \times 10^{-8}$ cm, $n = 10$, $(D_{10})_{air} = 5$ mm $= 0.5$ cm

Diameter of n^{th} dark ring is given by $D_n = \sqrt{\frac{4nR\lambda}{\mu}}$

For air film, $\mu = 1$ $\quad \therefore (D_n)^2 = 4nR\lambda$

Radius of curvature is -

$$\therefore R = \frac{(D_n)^2}{4n\lambda} = \frac{(0.5)^2}{4 \times 10 \times 5500 \times 10^{-8}}$$

$$= 113.63 \text{ cm}$$

To calculate diameter of 10^{th} dark ring when $\mu = 1.25$, use $D_n = \sqrt{\frac{4nR\lambda}{\mu}}$

$$\therefore D_{10} = \sqrt{\frac{4 \times 10 \times 113.63 \times 5500 \times 10^{-8}}{1.25}}$$

$$= 0.447 \text{ cm}$$

In a Newton's ring experiment, the diameter of the 10th dark ring changes from 1.4 cm to 1.27 cm when a liquid is introduced between the lens and the plate. Calculate the refractive index of the liquid.

Given: D_{10} (for air) = 1.4 cm, D_{10} (for liquid) = 1.27 cm

$$\frac{(D_n)_{air}}{(D_n)_{liquid}} = \sqrt{\mu}$$

Diameter of n^{th} dark ring is

$$(D_n)_{liquid} = \sqrt{\frac{4nR\lambda}{\mu}} \quad (1)$$

For air film
(refractive index = 1)

$$\therefore (D_n)_{air} = \sqrt{4nR\lambda} \quad (2)$$

Divide (2) by (1)

$$\frac{(D_n)_{air}}{(D_n)_{liquid}} = \sqrt{\mu}$$

$$\therefore \mu = \left(\frac{(D_n)_{air}}{(D_n)_{liquid}}\right)^2 = \frac{1.4^2}{1.27^2}$$

$$= 1.215$$

Electrodynamics

Electrodynamics is a branch of physics which deals with the study electric field, magnetic field and electromagnetic field.

- The charge at a point produces electric field,
- The charge in motion produce magnetic field and
- The variation of magnetic field produces emf.

The various quantities in electrodynamics like electric charge, magnetic field, charge density, electric flux, magnetic flux etc. are function of space and time. Therefore, to study laws of electrostatic and electrodynamics we required coordinate system and vector calculus.

There are three coordinate systems are used to specify a position of a point in a space.

1) **Cartesian coordinate system:**

Cartesian or Rectangular Coordinates (x, y, z)

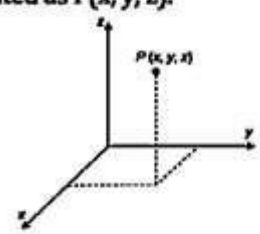
A point P in Cartesian coordinates is represented as P(x, y, z).

The ranges of coordinate variables are

$-\infty < x < \infty$

$-\infty < y < \infty$

$-\infty < z < \infty$



A vector $\vec{A}$ in the Cartesian coordinate system is written as,

$$\vec{A} = A_x\hat{\imath} + A_y\hat{\jmath} + A_z\hat{k}$$

Where, $\hat{\imath}, \hat{\jmath}$ and $\hat{k}$ are the unit vectors along the x, y and z directions respectively.

2) **Cylindrical coordinate system:**

Cylindrical Coordinates P (r, ϕ, z)

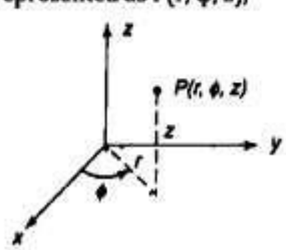
A point P in cylindrical coordinates is represented as P(r, ϕ, z),

The Ranges of Coordinates are,

$0 \le r < \infty$

$0 \le \emptyset < 2\pi$

$-\infty < z < \infty$



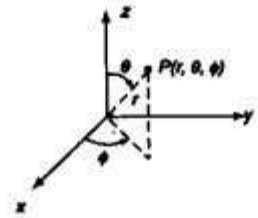
3) Spherical coordinate system:

Spherical or Polar Coordinates (r,θ,ø)

A point P in spherical coordinates is represented as P(r,θ,ϕ)

The ranges of coordinate variables are,

$$0 \le r < \infty$$
$$0 < \theta < \pi$$
$$0 \le \emptyset < 2\pi$$



Relation between Cartesian(x,y,z) and Spherical (r,θ,ϕ) Coordinates

$$r = \sqrt{x^2 + y^2 + z^2} \quad \theta = \tan^{-1}\left(\frac{\sqrt{x^2 + y^2}}{z}\right) \quad \phi = \tan^{-1}\left(\frac{y}{x}\right)$$

And

$$x = r\sin\theta\cos\phi \quad y = r\sin\theta\sin\phi \quad z = r\cos\theta$$

Relations between Cartesian (x, y, z) and Cylindrical (r, ø, z) Coordinates

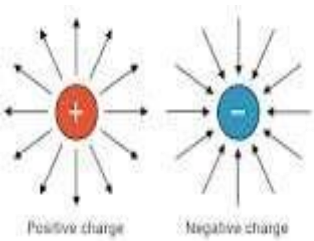
$$r = \sqrt{x^2 + y^2} \quad \phi = \tan^{-1}\left(\frac{y}{x}\right) \quad z = z$$

And

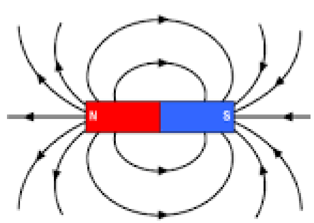
$$x = r\cos\phi \quad y = r\sin\phi \quad z = z$$

Q. Define a field? Explain scalar field and vector field?

Field of physical quantity:

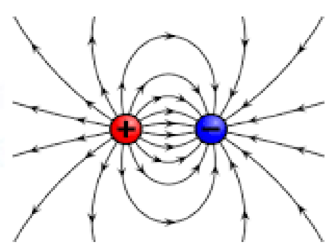


Electric field



Magnetic field

Gravitational field



Electric potential

- A field of physical quantity is the region in which physical quantity has a particular value at every point. E.g. Electric field, magnetic field, gravitational field, temperature around hot body, sound intensity in a room etc.

- A field is a function that describes the behavior of a physical quantity at all points in a given region of space.
- The physical quantity described by the field can be either a scalar or vector.
- Thus, depending upon the physical quantity be either scalar or vector, a field can also be a scalar field or vector field.

Scalar field:

- A region in which a scalar physical quantity has a particular value at every point is known as scalar field.
- Scalar field is specified by only magnitude.
- e.g. Temperature around hot body, Electric potential around charged body, Sound intensity in a room etc.

Vector field:

- A region in which a vector physical quantity has a particular value and direction at every point is known as vector field.
- Vector field is specified by magnitude as well as direction.
- e.g. Electric field, Magnetic field, Gravitational field etc.

Q. Explain DEL $\vec{}$ operator?

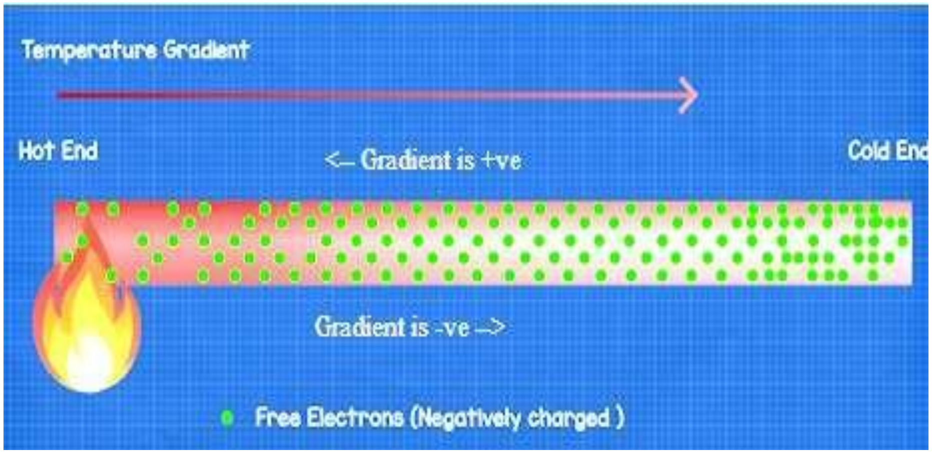
DEL $\vec{}$ operator

It is a vector differential operator used to study electrodynamics. A DEL operator is not a vector, but when it operates on scalar function, it becomes a vector. In Cartesian coordinate system Point 'P' has coordinate (x, y, z) and i, j, and k are unit vectors then DEL operator is given as

$$\vec{\nabla} = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

Q. Explain the physical significance of gradient of a scalar field?

The gradient of a scalar field:



1) Gradient is the rate of change of a physical quantity with distance.
2) When metal bar is heated at one end, the rate of change of temperature along the length of metal bar is known as temperature gradient.
3) The temperature is function of x, y, z in Cartesian coordinate system. Therefore T = T(x, y, z) is a scalar function.
4) Ÿ. T is called temperature gradient of a scalar function.
5) The gradient is rate of change of a function in a specified direction. Therefore, gradient of scalar function is a vector quantity.
6) The gradient is – ve when value of function is decreasing along the given direction and gradient is + ve when value of function is increasing along the given direction.

$$\vec{\nabla} = \hat{i}\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

$$\boxed{\vec{\nabla}T = \hat{i}\frac{\partial T}{\partial x} + j\frac{\partial T}{\partial y} + k\frac{\partial T}{\partial z}}$$

Ex: If $\emptyset(x,y,z) = 3(x^2y - y^2x)$, Calculate gradient.

Given:- $\emptyset(x,y,z) = 3(x^2y - y^2x)$

Formula:-

Solution:-

$$\text{Grad } \emptyset = \nabla\emptyset = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\{3(x^2y - y^2x)\}$$

$$= \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx) + \hat{k}(0)$$

Therefore, $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$

Ans:- The gradient is , $\nabla\emptyset = \hat{\imath}(6xy - 3y^2) + \hat{\jmath}(3x^2 - 2yx)$.

Find the gradient of $\phi(x,y,z) = 3x^2y - y^3z^2$ at point (1,-2,-1).

$$\frac{\partial\phi}{\partial x} = \frac{\partial(3x^2y - y^3z^2)}{\partial x} = 6xy \qquad \frac{\partial\phi}{\partial y} = \frac{\partial(3x^2y - y^3z^2)}{\partial y} = 3x^2 - 3y^2z^2$$

$$\frac{\partial\phi}{\partial z} = \frac{\partial(3x^2y - y^3z^2)}{\partial z} = -2y^3z$$

$$\overrightarrow{\nabla\phi} = \frac{\partial\phi}{\partial x}\hat{x} + \frac{\partial\phi}{\partial y}\hat{y} + \frac{\partial\phi}{\partial z}\hat{z}$$

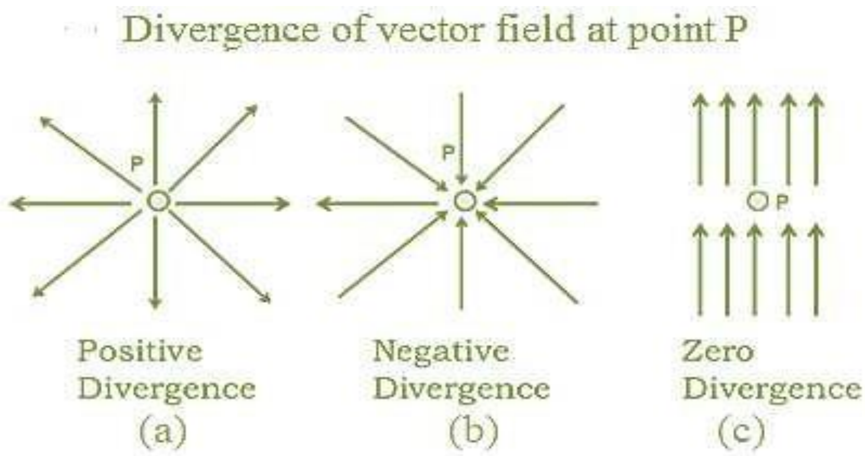
$$= (6xy)\,\hat{x} + (3x^2 - 3y^2z^2)\,\hat{y} - (2y^3z)\,\hat{z}$$

$$\therefore \left.\overrightarrow{\nabla\phi}\right|_{(1,-2,-1)} = 6(1)(-2)\,\hat{x} + (3(1)^2 - 3(-2)^2(-1)^2)\,\hat{y} - 2(-2)^3(-1)\,\hat{z}$$

$$= -12\,\hat{x} - 9\,\hat{y} - 16\,\hat{z}$$

Q. Explain the physical significance of divergence of vector field?

Divergence of vector field:



- The spreading or converging of a vector field from a certain point is known as divergence of vector field.
- In fig (a) the field is diverging from point ‘P’, therefore divergence is positive.

  In fig (b), the field is converging at point ‘P’, hence divergence is negative. In fig (c), the field is neither spreads nor converges, hence divergence is zero.

- Divergence of vector field is measure of how much vector field converges to or diverges form given point in volume. Divergence denotes only the magnitude of change and so, it is a scalar quantity.

- **When DEL operator acts on a vector function $\vec{V}$ = V (x, y, z), then $\vec{\nabla}.\vec{V}$ is called divergence of vector function.**

$$\overline{V} = v_x \hat{i} + v_y j + v_z \hat{i}$$

$$\overline{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\overline{\nabla}.\overline{V} = (\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}).(v_x \hat{i} + v_y j + v_z \hat{i})$$

$$\overline{\nabla}.\overline{V} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

Thus, divergence of a vector function is scalar.

If $\overrightarrow{\mathbf{v_A}} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$ and $\overrightarrow{\mathbf{v_B}} = y\,\hat{y}$, calculate their divergence.

$$\vec{\nabla}\cdot\overrightarrow{v_A} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot(x\hat{x} + y\hat{y} + z\hat{z})$$

$$\vec{\nabla}\cdot\overrightarrow{v_A} = \left(\frac{\partial x}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial z}{\partial z}\right) = 1 + 1 + 1 = 3$$

$$\vec{\nabla}\cdot\overrightarrow{v_B} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot(y\hat{y})$$

$$\vec{\nabla}\cdot\overrightarrow{v_B} = \left(\frac{\partial 0}{\partial x} + \frac{\partial y}{\partial y} + \frac{\partial 0}{\partial z}\right) = 0 + 1 + 0 = 1$$

If $\vec{\mathbf{v}} = \mathbf{1}\,\hat{x} + \mathbf{2}\,\hat{y} + \mathbf{3}\,\hat{z}$, find divergence of $\vec{\mathbf{v}}$.

$$\vec{\nabla}\cdot\vec{\mathbf{v}} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot(1\hat{x} + 2\hat{y} + 3\hat{z})$$

$$\vec{\nabla}\cdot\vec{\mathbf{v}} = \left(\frac{\partial(1)}{\partial x} + \frac{\partial(2)}{\partial y} + \frac{\partial(3)}{\partial z}\right)$$

$$= 0 + 0 + 0 = 0$$

If $\vec{\mathrm{A}} = x^2z\,\hat{x} - 2y^2z^2\,\hat{y} + xy^2z\,\hat{z}$, Find $\vec{\nabla}\cdot\vec{\mathrm{A}}$ at point (1,-1,1)

$$\vec{\nabla}\cdot\vec{A} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot(x^2z\,\hat{x} - 2y^2z^2\,\hat{y} + xy^2z\,\hat{z})$$

$$\vec{\nabla}\cdot\vec{A} = \left(\frac{\partial(x^2z)}{\partial x} + \frac{\partial(-2y^2z^2)}{\partial y} + \frac{\partial(xy^2z)}{\partial z}\right)$$

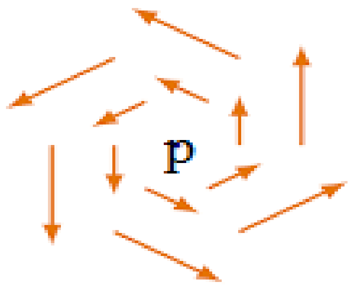
$$= 2xz - 4yz^2 + xy^2$$

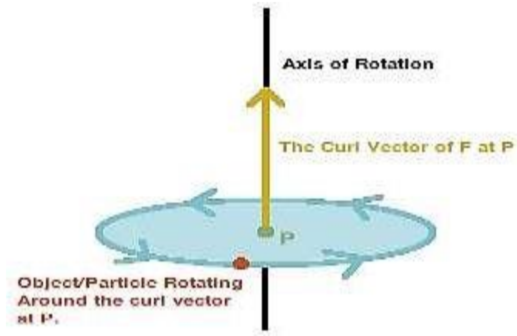
$$\vec{\nabla}\cdot\vec{A}\Big|_{(1,-1,1)} = 2(1)(1) - 4(-1)(1)^2 + (1)(-1)^2$$

$$= 2 + 4 + 1 = 7$$

Q. Explain the physical significance of curl of a vector field?

Curl of vector field:





1) The curl of vector field at any point gives how much the vector quantity curls or twist around that point.

   Consider water draining downs the sink; it will swirl in a rotational motion before going out. If we plot this rotational flow of water as vectors and measure it, it will denote the Curl. The length of the curl vector is a measure of how quickly particles move around the axis.

1) Since it has magnitude and direction, curl of a vector field is a vector quantity.
2) If curl of vector field is zero, the vector field is said to be irrotational.
3) Divergence of the curl of a vector is zero.
4) The curl of a vector field in Cartesian coordinate system is given by

$$\overrightarrow{V} = v_x \hat{i} + v_y j + v_z \hat{i}$$

$$\overrightarrow{\nabla} X \overrightarrow{V} = \begin{pmatrix} \hat{i}_x & \hat{i}y & \hat{i}z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{pmatrix}$$

$$\overrightarrow{\nabla} X \overrightarrow{V} = \hat{i}_x \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{i}y\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{i}z\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Q. Show that divergence of the curl of a vector is zero?

Divergence of the curl of a vector is zero:

Let us consider $\vec{v} = v_x\hat{x} + v_y\hat{y} + v_z\hat{z}$.

Curl of $\vec{v}$ is given by -
$$\vec{\nabla}\times\vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix}$$

$$\vec{\nabla}\times\vec{v} = \hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Divergence of Curl of $\vec{v}$ is given by -

$$\vec{\nabla}\cdot\left(\vec{\nabla}\times\vec{v}\right) = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right)\cdot\left(\hat{x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)\right)$$

$$= \frac{\partial}{\partial x}\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z}\right) + \frac{\partial}{\partial y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \frac{\partial}{\partial z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

$$= \left(\frac{\partial^2 v_z}{\partial x\partial y} - \frac{\partial^2 v_y}{\partial x\partial z}\right) + \left(\frac{\partial^2 v_x}{\partial y\partial z} - \frac{\partial^2 v_z}{\partial y\partial x}\right) + \left(\frac{\partial^2 v_y}{\partial z\partial x} - \frac{\partial^2 v_x}{\partial z\partial y}\right) = 0$$

Thus divergence of a curl is zero.

If $\vec{v} = x\,\hat{x} + y\,\hat{y} + z\,\hat{z}$, calculate its curl.

$$\vec{\nabla} \times \vec{v} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \times (x\,\hat{x} + y\,\hat{y} + z\,\hat{z})$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$\vec{\nabla} \times \vec{v} = \hat{x}\left(\frac{\partial z}{\partial y} - \frac{\partial y}{\partial z}\right) + \hat{y}\left(\frac{\partial x}{\partial z} - \frac{\partial z}{\partial x}\right) + \hat{z}\left(\frac{\partial y}{\partial x} - \frac{\partial x}{\partial y}\right)$$

$$= 0 + 0 + 0 = 0$$

Calculate curl of $\vec{v}$ if $\vec{v} = -y\hat{x} + x\hat{y}$.

$$\vec{\nabla} \times \vec{v} = \left(\hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}\right) \times (-y\,\hat{x} + x\,\hat{y} + 0\,\hat{z})$$

$$\vec{\nabla} \times \vec{v} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -y & x & 0 \end{vmatrix}$$

$$\vec{\nabla} \times \vec{v} = \hat{x}\left(\frac{\partial(0)}{\partial y} - \frac{\partial(x)}{\partial z}\right) + \hat{y}\left(\frac{\partial(-y)}{\partial z} - \frac{\partial(0)}{\partial x}\right) + \hat{z}\left(\frac{\partial(x)}{\partial x} - \frac{\partial(-y)}{\partial y}\right)$$

$$\vec{\nabla} \times \vec{v} = \hat{x}(0) + \hat{y}(0) + \hat{z}(1 - (-1)) = 2\hat{z}$$

Ex. Find the divergence and curl of the field $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$ in Cartesian coordinates.

Given:- $F = 30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k}$

Formula:-Divergence $(\vec{\nabla}.\vec{A})= \hat{\imath}\frac{\partial}{\partial x}A_x + \hat{\jmath}\frac{\partial}{\partial y}A_y + \hat{k}\frac{\partial}{\partial z}A_z$

Curl: $(\vec{\nabla} \times \vec{A}) = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$

Solution:-

Divergence, $\vec{\nabla}.\vec{F} = \left(\hat{\imath}\frac{\partial}{\partial x} + \hat{\jmath}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right).(30\hat{\imath} + 2xy\hat{\jmath} + 5xz^2\hat{k})$

$$= \frac{\partial}{\partial x}(30) + \frac{\partial}{\partial y}(2xy) + \frac{\partial}{\partial z}(2xz^2) = 2x + 10xz$$

$$\vec{\nabla}.\vec{F} = 2x(1 + 5z)$$

$$\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 30 & 2xy & 5xz^2 \end{vmatrix} = -5z^2\,\hat{\jmath} + 2y\,\hat{k}$$

Ans:- Divergence of field is $2x(1 + 5z)$ and its curl $= -5z^2\,\hat{\jmath} + 2y\,\hat{k}$

Q. State and explain Divergence and Stokes theorem used in electrodynamics?

1) Divergence theorem:

If V is the volume bounded by the surface S, then volume integral of the divergence of a vector function $\overline{\mathbf{V}}$ over volume V is equal to the surface integral of vector function $\overline{V}$ over the surface S enclosing the volume V.

$$\int_V \vec{\nabla}\cdot\vec{v}\ dv = \oint_S \vec{v}\cdot\overrightarrow{ds}$$

It is used to convert volume integral to surface integral and vice versa.

2) Stokes's theorem:

Is S is the surface area bounded by the boundary P, then surface integral of the curls of a vector function $\overline{\mathrm{V}}$ over surface area S is equal to the line integral of the vector function $\overline{\mathrm{V}}$ over the closed curve P bounding that surface area.

$$\int_S (\vec{\nabla}\times\vec{\mathrm{v}})\cdot ds = \oint_P \vec{\mathrm{v}}\cdot dl$$

It is used to convert a surface integral to a line integral and vice versa.

Static field & Time varying field:

Q. Explain the term static field and time varying field of physical quantity? Static field:

If the value of the physical quantity describing the field does not vary with time, then field is called static field. e.g. Static electric field accelerate the electrons and static magnetic field deflect the electrons used in CRT.

Time varying field:

If the value of physical quantity changes with time in the field region, then the field is called time varying field. e.g. Electromagnetic field.

Q. Derive the Maxwell's equation in differential (point) form and integral form? Give its physical significance?

Maxwell's Equation's:

- Maxwell's equations are a series of four partial differential equations on laws of electricity and magnetism that describe the force of electromagnetism.
- They were derived by mathematician James Clerk Maxwell.
- Gauss law for static electric field
- Gauss law for static magnetic field
- Faraday's law of electromagnetic field
- Ampere's law of magneto electric field
- These equations provide the mathematical background for the study of electromagnetic waves, transmission lines and antenna.

Steps for writing Maxwell's Equations

1. State the Law
2. Write the Law in Integral form
3. Use Divergence Theorem / Stokes Theorem
4. Write the Law in Differential Form

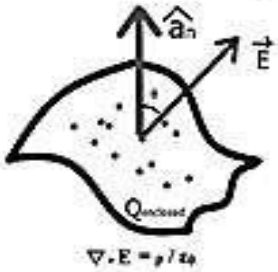
First Maxwell's Equation: (Gauss law for static electric field)

Statement: The total electric flux through a closed surface is $1/\epsilon_0$ times the total charge enclosed inside the closed surface.

Total electric flux $= 1/\epsilon_0$ X Total charge enclosed

$$\Phi = \frac{1}{\epsilon_0}\sum_{i=1}^{n} q_i$$

Consider an irregular surface of any shape enclosed number of +ve charges.



Total flux through a closed surface is given by surface integral of electric field intensity.

$$\phi = \int_s \vec{E}\,\vec{ds}$$

Total charge enclosed by the closed surface is given by volume integral of the charge density.

$$Q = \int_v \rho \; dv$$

Therefore $\int_s \vec{E}\,\vec{ds} = \frac{1}{\epsilon_0}\int_v \rho \; dv$ This is integral form of Gauss law in static electric field. This is integral form of Maxwell's first equation.

Integral form of Gauss law in static electric field is

$$\int_s \vec{E}\,\vec{ds} = \frac{1}{\epsilon_0}\int_v \rho \; dv$$

Applying divergence theorem, the surface integral is converted in to volume integral.

$$\int_s \vec{E}\;\vec{ds} = \int_v \nabla . \vec{E} \; dv$$

$$\int_v \nabla . \vec{E} \; dv = \frac{1}{\epsilon_0}\int_v \rho \; dv$$

$$\vec{\nabla} . \vec{E} = \frac{\rho}{\epsilon_0}$$

The divergence of electric field is equal to $1/\epsilon_0$ times the charge density inside a closed surface.

This is differential or point form of Gauss law in static electric field. This is differential form of Maxwell's first equation.

Physical significance Maxwell's first equation:

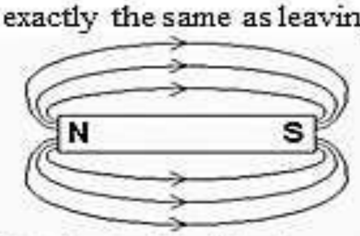
The divergence of electric field is equal to $1/\epsilon_0$ times the charge density inside a closed surface.

$$\overline{\nabla} \cdot \overline{E} = \frac{\rho}{\epsilon_0}$$

1) Gauss law in static electric field gives relation between charge distribution and electric field.
2) It gives the relation between the distribution of charge and electric field produced. The divergence is the how much electric field spreads out from a given point.
3) If there is more charge enclosed by the surface, the divergence of electric field is more. If there is no charge, divergence is zero.

Second Maxwell's Equation: (Gauss law for static magnetic field)

Statement: "The net magnetic flux through a closed surface is zero. As no isolated magnetic pole exist, the number of magnetic lines of flux entering any surface is exactly the same as leaving from that surface"



Total outgoing flux is zero.

$$\oint_S \vec{B}\ ds = 0$$

This is integral form of Gauss law in static magnetic field. This is integral form of Maxwell's second equation.

Using divergence theorem, convert surface integral in to volume integral.

Therefore $\oint_S \vec{B}\ ds = \int_V \vec{\nabla} \cdot \vec{\mathrm{B}}\ dv$ $\qquad \vec{\nabla} \cdot \vec{\mathrm{B}} = 0$

The divergence of magnetic field is zero.

This is differential or point form of Gauss law in static magnetic field. This is differential form of Maxwell's second equation.

Physical significance Maxwell's second equation:

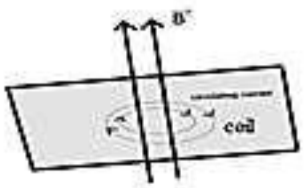
The divergence of magnetic field is zero.

$$\vec{\nabla} \cdot \vec{B} = 0$$

1) The magnetic field lines cannot start or end at any point. Isolated magnetic monopoles cannot exist.
2) Gauss' law for magnetism says that magnetic monopoles do not exist. Charges exist as positive or negative. But in magnetism, whenever you have a south pole, you also have a north pole - there are no single, or monopoles, as yet discovered.

Third Maxwell equation: (Faradays law of electromagnetic induction)

Statement: "Whenever there is change in the magnetic flux associated with a coil, an emf is induced in the coil. The magnitude of induced emf is directly proportional to the –ve rate of change of magnetic flux through the coil".



$$e = -\frac{d\phi}{dt}$$

Induced emf is the line integral of induced electric field around the length of the coil.

$$e = \int_l \overline{E}\ \overline{dl}$$

Total magnetic flux through the coil is the surface integral of magnetic flux density

$$\phi = \int_s \overline{B}\ \overline{ds}$$

$$\int_l \overline{E}\ \overline{dl} = -\frac{d}{dt}\int_s \overline{B}\ \overline{ds}$$

$$\int_l \overline{E}\ \overline{dl} = -\int_s \frac{\delta B}{\delta t}\ \overline{ds}$$

This is integral form of Maxwell's third equation.

Using Stokes theorem, convert line integral to surface integral.

$$\int_l \overline{E}\ \overline{dl} = \int_s (\overline{\nabla} \times \overline{E})\ \overline{ds}$$

$$\int_s (\overline{\nabla} \times \overline{E})\ \overline{ds} = -\int_s \frac{\delta \overline{B}}{\delta t}\ \overline{ds}$$

$$\overline{\nabla} \times \overline{E} = -\frac{\delta B}{\delta t}$$

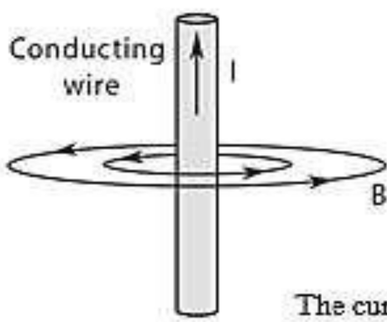
This is differential form of Maxwell's third equation.

Physical significance Maxwell's third equation:

1) It gives the relation between an electric field and changing magnetic flux.
2) Faraday's law says that any change to the magnetic environment of a coil of wire will cause a voltage to be induced in the coil. If the magnetic field strength changes, or the magnet moves, or the coil moves, or the coil is rotated - any of these things will create a voltage in the coil.

Fourth Maxwell equation: (Ampere circuital law for static magnetic field)

Statement: "The line integral of magnetic field intensity H around a closed path is equal to current enclosed by that path."



$$\int_l \overline{H}\ \overline{dl} = I$$

$$B = \mu_0 H$$

$$H = \frac{B}{\mu_0}$$

$$\int_l \overline{B}\ \overline{dl} = \mu_0 I$$

The current I is surface integral of current density J.

$$I = \int_s \overline{J}\ \overline{ds}$$

$$\int_l \overline{B}\ \overline{dl} = \mu_0 \int_s \overline{J}\ \overline{ds}$$

This is integral form of Ampere circuital law in static magnetic field. This is integral form of Maxwell's fourth equation.

Using Stokes theorem, convert line integral in to surface integral.

$$\int_s (\overline{\nabla} \times \overline{B})\, \overline{ds} = \mu_0 \int_s \overline{J}\, \overline{ds}$$

$$\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$$

This is differential form of Ampere circuital law in static magnetic field. This is differential form of Maxwell's fourth equation.

Physical significance Maxwell's fourth equation:

1) It gives the relation between electromagnetic field and electric current.
2) Ampere's law says that the magnetic field created by an electric current is proportional to the magnitude of that electric current, with a constant of proportionality equal to the permeability of free space.
3) Stationary charges produce electric fields, proportional to the magnitude of that charge. But moving charges produce magnetic fields, proportional to the current.

Applications of Maxwell's Equations

Wherever we have electromagnetic devices then we must remember Maxwell's equations.

- Antennas used to receive electromagnetic signals.
- Transmission lines.
- The ceiling fan capacitor works on displacement current, which is the introduction by Maxwell.
- MRI scanners in hospitals
- Magnetic tape
- Electricity generation

QUANTUM PHYSICS

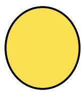
(Prerequisites: Dual nature of radiation, Photoelectric effect Matter waves-wave nature of particles, de-Broglie relation, Davisson-Germer experiment)

De Broglie hypothesis of matter waves; properties of matter waves; wave packet, phase velocity and group velocity; Wave function; Physical interpretation of wave function; Heisenberg uncertainty principle; nonexistence of electron in nucleus; Schrodinger's time dependent wave equation; time independent wave equation; Particle trapped in one dimensional infinite potential well, Quantum Computing.

Quantum physics is a branch of physics which deals with the properties and interaction of the matter.

According to scientist De'Broglie, matter possess dual nature i.e. wave and particle nature.

<u>Particle:-</u> The constituents of matter like electrons, protons, atoms, molecules are referred as particles.

<u>Properties:</u>- 

1. Particle has fixed mass.

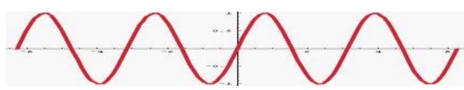
2. Particle can locate at definite point.

3. Particle has ability to move from one place to another place.

4. Particle release energy when suddenly stopped.

5. Particle is specified by mass, velocity, momentum and energy.

<u>Wave:-</u> The oscillatory disturbance produced by a particle is known as wave.



<u>Properties:</u>-

1. Wave spread out over a large region of space,

2. Wave cannot locate at any particular point.

3. Wave is specified by frequency, wavelength. Amplitude and phase.

Q. State and explain De 'Broglie's hypothesis?

De'Broglies hypothesis of matter wave:

According to scientist De 'Broglie, a particle of mass 'm' moving with velocity 'V' is associate with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$ Where p = mv is momentum of the particle.

The wave associated with the moving particle is known as De 'Broglie's wave or matter wave and wavelength is known as De 'Broglie's wavelength.

<u>Explanation:</u>- The radiation i.e photons has dual nature.

The phenomenon like interference, diffraction, polarization, reflection, refraction can only be explained if the radiation is to be considering as a wave.

The phenomenon like photoelectric effect, Compton Effect can only be explained if the radiation is to be consider as a particle.

If we consider a photon as a wave of frequency γ, then its energy is given by E = h γ

If we consider a photon as a particle of mass m, then its energy is given by E = mc^2

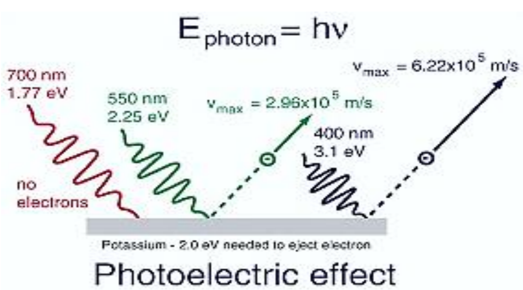
Therefore $mc^2 = h\gamma$ but $\gamma = c / \lambda$

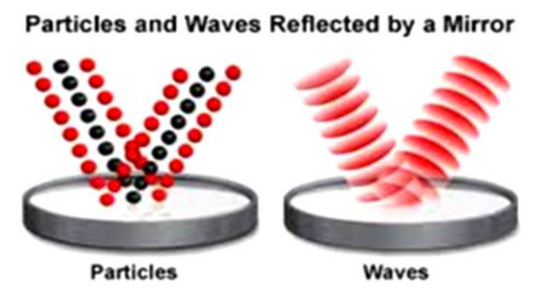
$$mc^2 = h\frac{c}{\lambda}$$

$mc = \frac{h}{\lambda}$ but mc = p momentum of photon

$p = \frac{h}{\lambda}$ therefore $\lambda = \frac{h}{p}$ this is wavelength of light wave.

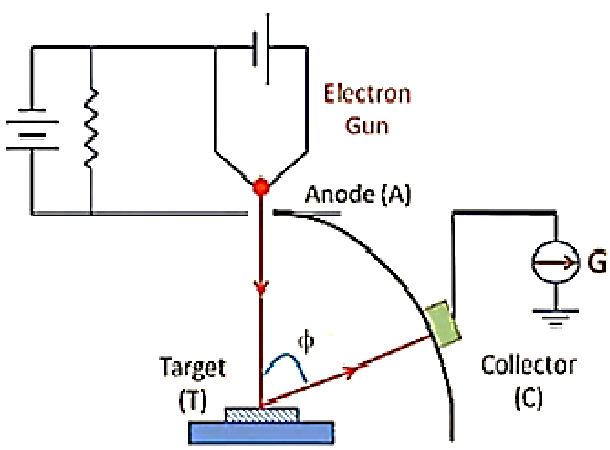
Similarly, if a particle of mass m moving with velocity v, then a wave of wavelength $\lambda = \frac{h}{mv}$ is associated with this particle.



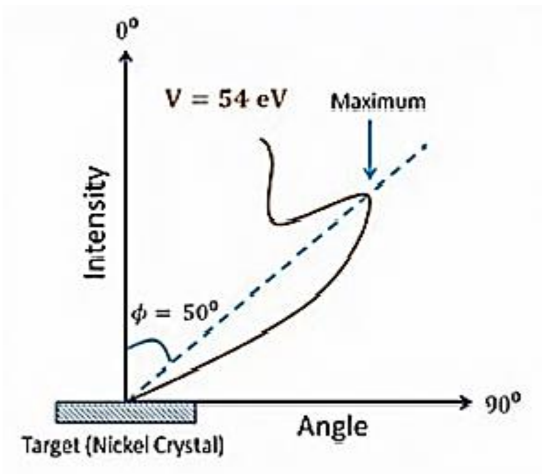


Experimental verification of D'Broglie's Hypothesis:

Davission & Germer experiment:



Theoretical value of d' Broglie's wavelength of an electron accelerated by potential 54 V is

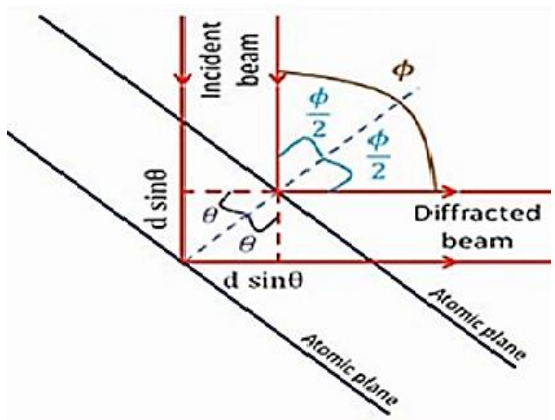


$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

$$\therefore \lambda = \frac{6.626 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$$\boxed{\therefore \lambda = 1.67 \times 10^{-10} \text{m} = 1.67 \text{ A}^0}$$

Practical value of d' Broglie's wavelength of an electron is calculated using Bragg's law



From Bragg's diffraction law, we have

$$2 \text{ d} \sin\theta = n\lambda$$

$$\therefore \lambda = \frac{2 \text{ d} \sin\theta}{n}$$

$$\therefore \lambda = \frac{2 \times 9.086 \times 10^{-11} \times \sin 65}{1}$$

$$\boxed{\therefore \lambda = 1.65 \times 10^{-10} \text{m} = 1.65 \text{ A}^0}$$

Thus wavelength of electron calculated using d' Broglie's relation & Bragg's law are same, therefore electron beam has wave nature.

Properties of matter wave:

(1) Matter wave represents the probability of finding a particle in space.

(2) Matter waves are not electromagnetic in nature.

(3) de-Brogile or matter wave is independent of the charge on the material particle. It means, matter wave of de-Broglie wave is associated with every moving particle (whether charged or uncharged).

(4) Practical observation of matter waves is possible only when the de-Broglie wavelength is of the order of the size of the particles.

(5) Electron microscope works on the basis of de-Broglie waves.

(6) The phase velocity of the matter waves can be greater than the speed of the light.

(7) Matter waves can propagate in vacuum, hence they are not mechanical waves.

(8) The number of de-Broglie waves associated with nth orbital electron is n.

(9) Only those circular orbits around the nucleus are stable whose circumference is integral multiple of de-Broglie wavelength associated with the orbital electron.

Q. Derive De' Broglie's wavelength of electron in terms of energy?

De' Broglie's wavelength of electron in terms of energy E:

Consider an electron of mass 'm' moving with velocity 'v', then energy of electron is

$$E = \frac{1}{2} mv^2$$

$mv^2 = 2E$ multiply both side by m

$m^2v^2 = 2mE$ but mv =P momentum of electron.

$$P^2 = 2mE$$

$$P = \sqrt{2mE}$$

De' Broglie wavelength is given by $\lambda = \frac{h}{p}$

Therefore $\lambda = \frac{h}{\sqrt{2mE}}$

Q. State De' Broglie's hypothesis & Show that De' Broglie's wavelength of electron is inversely proportional to square root of accelerating potential?

De' Broglie's wavelength of electron in terms accelerating potential Va:

If an electron of mass 'm' moving with velocity 'v' is accelerated by potential Va, then energy of electron in terms of Va is given by

$E = e\,Va$ ------(1) but $E = \frac{1}{2} mv^2$ --------------------- (2)

Equating equation (1) and (2)

$$\frac{1}{2} mv^2 = eVa$$

$mv^2 = 2\,e\,Va$ multiply both side by 'm'

$$m^2v^2 = 2\,meVa$$

$$P = \sqrt{2meVa}$$

De'Broglie wavelength is given by $\lambda = \frac{h}{p}$

Therefore $\lambda = \frac{h}{\sqrt{2meVa}}$

Where h = 6.63 X 10^{-34} J-S

m = 9.1 X 10^{-31} kg

e = 1.6 X 10^{-19} C

Therefore $\lambda = \frac{12.25}{\sqrt{Va}}$ AU

$$\lambda \ \alpha \ \frac{1}{\sqrt{Va}}$$

Thus De'Broglie's wavelength is inversely proportional to square root of accelerating potential.

An electron and photon each have a wavelength of 2 A^0. What are their momentum and energy? Mass of electron = 9.1×10^{-31}kg, Planck's constant = 6.63×10^{-34}J − s

Given: Mass of electron = 9.1×10^{-31}kg, Planck's constant = 6.63×10^{-34}J.s, $\lambda = 2 \times 10^{-10}$m

For an electron,

$$\text{momentum}, p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}} = 3.315 \times 10^{-24} \frac{kg-m}{s}$$

$$\text{Energy}, E = \frac{p^2}{2m} = \frac{(3.315 \times 10^{-24})^2}{2 \times 9.1 \times 10^{-31}} = 6.038 \times 10^{-18} J$$

For a photon,

$$\text{momentum}, p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}}{2 \times 10^{-10}} = 3.315 \times 10^{-24} \frac{kg-m}{s}$$

$$\text{Energy}, E = h\nu = \frac{hc}{\lambda} = \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{2 \times 10^{-10}} = 9.945 \times 10^{-16} J$$

An electrons accelerated through 100 volts are reflected from a crystal. Calculate the glancing angle at which the first order reflection occurs. Given lattice spacing = 2.15 A^0.

Given : $h = 6.63 \times 10^{-34}$J. s, $m = 9.1 \times 10^{-31}$kg, $d = 2.15 \times 10^{-10}$m

$e = 1.6 \times 10^{-19}$C, V = 100 volts, n = 1,

$\lambda = ?$ $\theta = ?$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}} = 1.229 \times 10^{-10} m$$

Using Bragg's law, $2 d \sin\theta = n\lambda$

$$\therefore \theta = \sin^{-1}\left(\frac{n\lambda}{2 d}\right) = \sin^{-1}\left(\frac{1 \times 1.229 \times 10^{-10}}{2 \times 2.15 \times 10^{-10}}\right) = 16.61^0$$

Find the energy of neutron in units of electron volt whose de Broglie wavelength is 1A^0. Given – mass of neutron = $1.674 \times 10^{-27} kg$

Given : $h = 6.63 \times 10^{-34}$ J – s, $m = 1.674 \times 10^{-27}$kg

$\lambda = 1\ A^0 = 10^{-10}$m, $E = ?$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} \qquad \therefore E = \frac{h^2}{2m\lambda^2} = \frac{(6.63 \times 10^{-34})^2}{2 \times 1.674 \times 10^{-27} \times (10^{-10})^2}$$

$$= 1.313 \times 10^{-20}\ J = \frac{1.313 \times 10^{-20}}{1.6 \times 10^{-19}} = 0.082\ eV$$

An electron accelerated through 1000 volts and is reflected from a crystal. The first order reflection occurs when glancing angle is 70^0. Calculate the interplanar spacing of the crystal.

Given : $h = 6.63 \times 10^{-34}$ J – s, $m = 9.1 \times 10^{-31}$kg,

$e = 1.6 \times 10^{-19}$C, $V = 1000$ volts,

$n = 1$, $\lambda = ?$ $d = ?$

$$\lambda = \frac{h}{\sqrt{2meV}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 1000}} = 3.885 \times 10^{-11}m$$

Using Bragg's law, $2\ d \sin\theta = n\lambda$

$$\therefore d = \frac{n\lambda}{2 \sin\theta} = \frac{1 \times 3.885 \times 10^{-11}}{2 \times \sin 70}$$

$$= 2.067 \times 10^{-11}m = 0.2067\ A^0$$

Calculate de Broglie wavelength associated with an α – particle accelerated by a potential difference of 100 kV. Mass of α – particle is 6.68×10^{-27}kg.

Given : $h = 6.63 \times 10^{-34}$J – s, $m = 6.68 \times 10^{-27}$kg,

$q = 2\ e = 2 \times 1.6 \times 10^{-19}$C, $V = 100 \times 10^3$ volts,

$\lambda = ?$

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mqV}} = \frac{6.63\times10^{-34}}{\sqrt{2\times6.68\times10^{-27}\times2\times1.6\times10^{-19}\times100\times10^3}}$$

$$= 3.206 \times 10^{-14}m$$

Calculate velocity and de Broglie wavelength of an α – particle of energy 1 keV. Mass of α – particle is 6.68 x 10^{-27} kg.

Given : $h = 6.63 \times 10^{-34}$J – s, $m = 6.68 \times 10^{-27}$kg

$E = 1$ keV $= 1000 \times 1.6 \times 10^{-19}$ J $v = ?$ $\lambda = ?$

For calculating velocity, $\frac{1}{2}mv^2 = E$

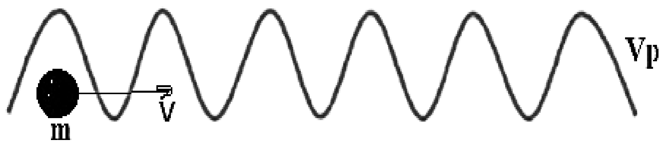
$$\therefore v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\times1000\times1.6\times10^{-19}}{6.68\times10^{-27}}} = 2.188 \times 10^5 m/s$$

For Wavelength, $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63\times10^{-34}}{\sqrt{2\times6.68\times10^{-27}\times1000\times1.6\times10^{-19}}}$

$$= 4.535 \times 10^{-13} m$$

Q. Explain the term phase velocity and group velocity?

Phase velocity (V_{ph}) :-



According to scientist De'Broglie, a particle of mass 'm' moving with velocity 'V' is associate with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

Where p = mv is momentum of the particle.

The wave associated with the moving particle is known as De'Broglie's wave or matter wave and wavelength is known as De'Broglie's wavelength.

The velocity with which the De'Broglie's wave associated with the moving particle is travel is known as phase velocity.

The equation of travelling sinusoidal wave is given by

$$Y = A\,sin\,(\omega t - kx)$$

Phase velocity $Vph = \frac{\omega}{k}$

But angular frequency $\omega = 2\pi\gamma$ and wave number $k = \frac{2\pi}{\lambda}$

Therefore $Vph = \gamma \times \lambda$

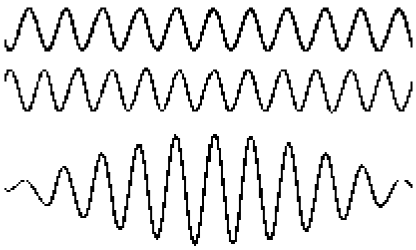
but $E = h\gamma$ and $E = mc^2$

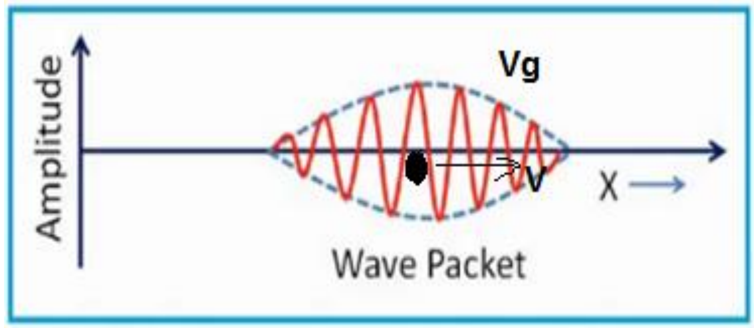
$mc^2 = h\gamma$

$\gamma = \frac{mc2}{h}$ and De'Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$

Therefore, $Vph = \frac{mc2}{h} \times \frac{h}{mv}$ hence $\boxed{Vph = \frac{c2}{v}}$

Group velocity (Vg):-





According to scientist De'Broglie, a particle of mass 'm' moving with velocity 'V' is associated with a wave of wavelength $\lambda = \frac{h}{mv} = \frac{h}{p}$

Where p = mv is momentum of the particle.

The wave associated with the moving particle is known as De'Broglie's wave or matter wave and wavelength is known as De'Broglie's wavelength.

The particle is moving with velocity 'V' and wave is moving with velocity c^2/v is not possible, therefore wave associated with the moving particle is not a continuous wave, but it is a group of waves formed by superposition of individual wave of different frequency. This group of waves is known as wave group or wave packet.

The phases and amplitude of the waves in group are such that they undergo constructive interference only over a small region where the particle may be located.
Outside this region, destructive interference occurs and amplitude is zero.
Velocity with which this wave packet moves is called as group velocity. This group velocity is equal to the velocity of particle.

The velocity with which entire wave group travel is known as group velocity. It is the velocity with which the energy transmission takes place in a wave.

The equation of travelling sinusoidal wave with slightly modulated w and k is given by

$$Y1 = A \sin(\omega t - kx)$$

$$Y2 = A \sin(\omega + dw)t - (k + dk)x)$$

The resultant is Y = Y1 + Y2

The group velocity is given by $Vg = \frac{d\omega}{dk}$

Q. Show that group velocity = particle velocity?

Expression for group velocity = particle velocity:

The group velocity is given by $Vgroup = \frac{d\omega}{dk}$ -------------- (1)

We have angular frequency $\omega = 2\pi\gamma$ and wave number $k = \frac{2\pi}{\lambda}$

but E = h γ and E = mc^2

mc^2 = h γ

Therefore $\gamma = \frac{\text{mc2}}{h}$ and De'Broglie wavelength is given by $\lambda = \frac{h}{p} = \frac{h}{mv}$

$$\omega = \frac{2\pi\text{mc2}}{h} \quad \text{and} \quad k = \frac{2\pi mv}{h}$$

The relativistic expression for mass of the particle is given by

$$m = mo\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\omega = \frac{2\pi m_0 c^2}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} \quad \text{and} \quad k = \frac{2\pi mov}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}}$$

$$\frac{d\omega}{dv} = \frac{2\pi m_0 c^2}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{1}{2}} . -\frac{1}{2}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} . \frac{-2v}{c^2}$$

$$= \frac{2\pi\text{mov}}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}} . \text{ ------------- (2)}$$

Similarly $\frac{dk}{dv} = \frac{2\pi\text{mo}}{h}\left(1 - \frac{v^2}{c^2}\right)^{-\frac{3}{2}}$ -------------- (3)

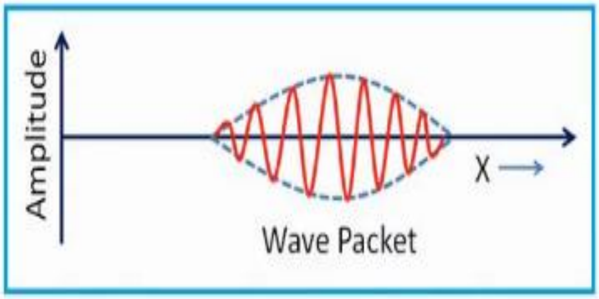
Divide eqn. (2) by eqn. (3)

$$\frac{d\omega}{dk} = v$$

Thus group velocity = particle velocity

Q. State and explain Heisenberg's uncertainty principle? Give its one experimental illustration?

Heisenberg's uncertainty principal:-



A wave group is associated with the moving particle and energy transmission takes place in the form of wave group. When wave group is small, we can easily locate the position of the particle but since small wave group contains number of waves, it is difficult to determine the wavelength i.e momentum of the particle. When wave group is large, we can easily determine the wavelength i.e momentum of the particle but since particle may be anywhere within a large wave group, it is difficult to locate the position of the particle.

Thus certainty in the measurement of position of particle gives uncertainty in the measurement of momentum of the particle and vice versa. Thus it is not possible to measure the position and momentum of particle simultaneously and precisely.

<u>Statement</u>:- "It is impossible to measure the position and momentum of particle simultaneously and precisely. The product of uncertainties in the measurement of position and momentum of the particle with in wave group is greater than or equal to planks constant".

$$\Delta x . \Delta p \geq h$$

More precisely,

$$\Delta x . \Delta p \geq \hbar \quad \text{where } \hbar = \frac{h}{2\pi} = 1.055 \text{ X } 10^{-34}$$

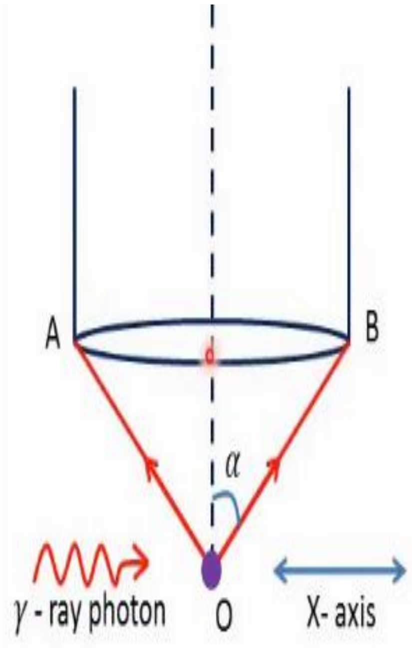
This principle is valid for any two related physical quantities,

$$\Delta E . \Delta t = \frac{h}{4\pi}$$

$$\Delta L . \Delta \theta = \frac{h}{4\pi}$$

Experimental illustration of Heisenberg's uncertainty principle:-

1) Scattering of photon with in a microscope:-



Consider a photon of energy E = hυ is incident on a particle and scatter with in a microscope. The photon is scatter along X- axis.

If Δx is uncertainty in the measurement of position of photon.

$$2\,\Delta x \sin\alpha = \lambda$$

$$\Delta x = \frac{\lambda}{2\sin\alpha} \text{ --------- (1)}$$

The X- component of momentum lies between P sin α and – P sin α.

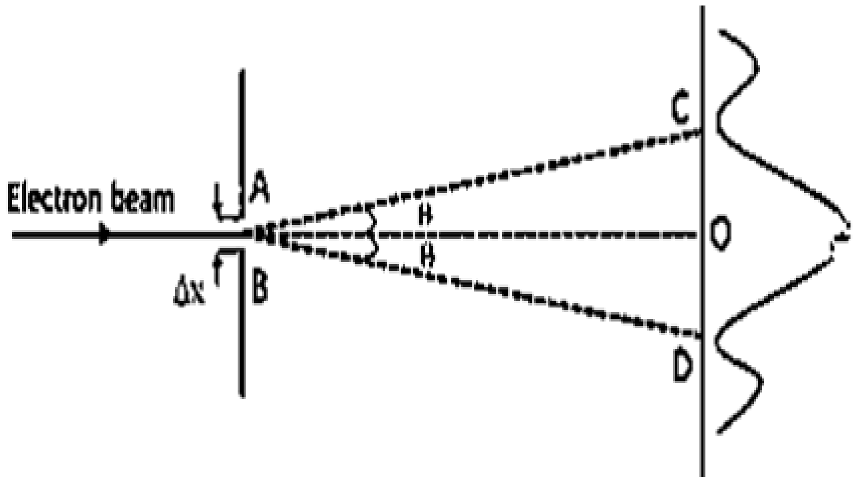
Therefore uncertainty in the measurement of momentum of photon is

$$\Delta P = P\sin\alpha - (-P\sin\alpha) = 2P\sin\alpha \text{ -------------- (2)}$$

From equation (1) and (2) $\Delta x\,\Delta P = \frac{\lambda}{2\sin\alpha} \times 2P\sin\alpha = \lambda P$

But $\lambda = \frac{h}{p}$ therefore $\Delta x\,\Delta P = h$

Diffraction of electrons by the single slit:-



Consider single slit of width 'd'= Δy is illuminated by the electrons. The electrons are diffracted by the slit along Y-axis and produce diffraction pattern on the photographic plate. The central maximum is formed at P_0 and secondary maxima and minima are obtained on both sides of central maxima.

The uncertainty in the measurement of position of electron = slit width = Δy

$$2\,\Delta y \sin\theta = \lambda$$

$$\Delta y = \frac{\lambda}{2\sin\theta} \text{ --------- (1)}$$

The Y- component of momentum lies between Py sinθ and – Py sinθ.

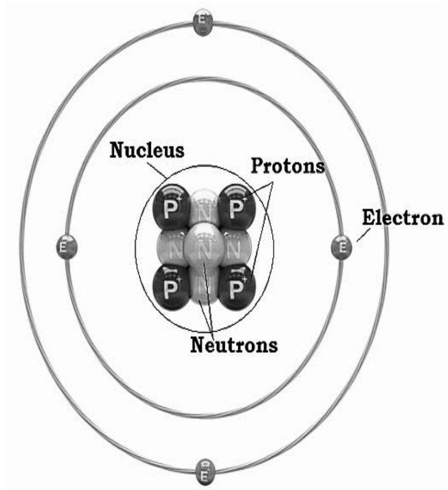
Therefore uncertainty in the measurement of momentum of photon is

$$\Delta Py = Py\sin\theta - (-Py\sin\theta) = 2\,Py\sin\theta \text{ -------------- (2)}$$

From equation (1) and (2) $\Delta y\,\Delta Py = \frac{\lambda}{2\sin\theta} \text{ X } 2Py\sin\theta = \lambda Py$

But $\lambda = \frac{h}{p}$ therefore $\Delta y\,\Delta Py = h$

Q. State Heisenberg's uncertainty principle? Show that electron cannot exist in the nucleus of an atom?



Diameter of nucleus of an atom is 10^{-14} m.

Uncertainty in the measurement of position of electron with in nucleus is

$$\Delta x = 10^{-14}\ \text{m}$$

According to HUP, $\Delta x . \Delta p \geq \hbar$ Where $\hbar = \frac{h}{2\pi} = 1.055 \text{ X } 10^{-34}$

$$\Delta p = \frac{1.055 \text{ X } 10^{-34}}{\Delta x}$$

$$\Delta p = \frac{1.055 \text{ X } 10^{-34}}{10^{-14}}$$

$$\Delta p = 10^{-20} Kg.m/sec$$

The energy of an electron is E = $\Delta p\, .\, c = 10^{-20} . 3X\ 10^{8} = 3\ X\ 10^{-12} J$

$$E = \frac{3\ X\ 10^{-12}}{1.6\ X\ 10^{-19}}\ eV = 19\ MeV$$

Thus energy required for electron to remain inside the nucleus is 19 MeV. It is not possible, therefore electron never exist inside the nucleus.

Alternate solution:

$\therefore$Uncertainty in position if electron exist in nucleus = $\Delta x = 2r$

$$\therefore \Delta x = 2r \approx 10^{-14} m$$

According to HUP $\boxed{\Delta x \, \Delta p \geq \frac{h}{4\pi}}$

$$\therefore \Delta x \, m\Delta v \geq \frac{h}{4\pi}$$

$$\therefore \Delta v \geq \frac{h}{4\pi \, m \, \Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 10^{-14}}$$

$$\therefore \Delta v \geq 5.797 \times 10^{9} \ m/s$$

Thus velocity of electron required is greater than velocity of light, this is not possible. Therefore electron never exists inside the nucleus.

Numerical:

An electron confined in a box of length 10^{-8} m. Calculate minimum uncertainty in its velocity.

Given : $\Delta x = 10^{-8} m$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34}$, $\Delta v = ?$

According to HUP, $\Delta x . \Delta p = \Delta x . m \, \Delta v \geq \frac{h}{4\pi}$

$$\therefore \Delta v \geq \frac{h}{4\pi} \times \frac{1}{m \, \Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 10^{-8}}$$

$$\therefore \Delta v = 5797 \ m/s$$

The speed of an electron is measured to within an uncertainty of 2×10^4 m/s. What is the minimum space required by the electron to be confined to an atom?

Given : $\Delta v = 2 \times 10^4 \frac{m}{s}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34} \ J - s$, $\Delta x = ?$

According to HUP, $\Delta x \, \Delta p = \Delta x \, m \, \Delta v \geq \frac{h}{4\pi}$

$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{m \, \Delta v} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 9.1 \times 10^{-31} \times 2 \times 10^{4}}$$

$$\therefore \Delta x = 2.898 \times 10^{-9} m$$

An electron has a speed of 900 m/s with an accuracy of 0.001%. Calculate the uncertainty in the position of the electron.

Given : $v = 900\frac{m}{s}$, $\frac{\Delta v}{v} = \frac{0.001}{100}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34}$ J − s, $\Delta x = ?$

$$p = m\,v = 9.1 \times 10^{-31} \times 900 = 8.19 \times 10^{-28}\ \frac{kg.m}{s}$$

$$\Delta p = m\,\Delta v = m\,v\,\frac{\Delta v}{v}$$

$$= p \times \frac{\Delta v}{v} = 8.19 \times 10^{-28} \times \frac{0.001}{100} = 8.19 \times 10^{-33}\ \frac{kg.m}{s}$$

According to HUP, $\Delta x. \Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{\Delta p} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 8.19 \times 10^{-33}} = 6.44 \times 10^{-3} m$$

An electron has a speed of 400 m/s with uncertainty of 0.01%. Find the accuracy in its position.

Given : $v = 400\frac{m}{s}$, $\frac{\Delta v}{v} = \frac{0.01}{100}$, $m = 9.1 \times 10^{-31}$ kg,

$h = 6.63 \times 10^{-34}$, J − s $\Delta x = ?$

$$p = m\,v = 9.1 \times 10^{-31} \times 400 = 3.64 \times 10^{-28}\ \frac{kg.m}{s}$$

$$\Delta p = m\,\Delta v = m\,v\,\frac{\Delta v}{v}$$

$$= p \times \frac{\Delta v}{v} = 3.64 \times 10^{-28} \times \frac{0.01}{100} = 3.64 \times 10^{-32}\ \frac{kg.m}{s}$$

According to HUP, $\Delta x. \Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta x \geq \frac{h}{4\pi} \times \frac{1}{\Delta p} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 3.64 \times 10^{-32}} = 1.449 \times 10^{-3} m$$

A position and momentum of 1 keV electron are simultaneously measured. If position is located within 10 nm then what is the percentage uncertainty in its momentum?

Given : $E = 1\ keV = 1000 \times 1.6 \times 10^{-19}$ J, $\Delta x = 10 \times 10^{-9} m$

$\frac{\Delta p}{p} \times 100 = ?$

$$p = \sqrt{2mE} = \sqrt{2 \times 9.1 \times 10^{-31} \times 1000 \times 1.6 \times 10^{-19}} = 1.706 \times 10^{-23}\ \frac{kg.m}{s}$$

According to HUP, $\Delta x. \Delta p \geq \frac{h}{4\pi}$

$$\therefore \Delta p \geq \frac{h}{4\pi} \times \frac{1}{\Delta x} = \frac{6.63 \times 10^{-34}}{4 \times 3.142 \times 10 \times 10^{-9}}$$

$$\therefore \Delta p = 5.275 \times 10^{-27}\ \frac{kg.m}{s}$$

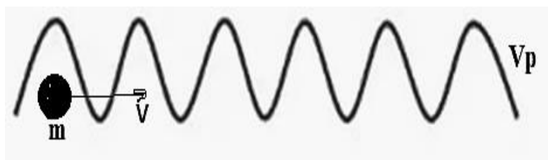
$$\therefore \frac{\Delta p}{p} \times 100 = \frac{5.275 \times 10^{-27}}{1.706 \times 10^{-23}} \times 100 = 0.0309\ \%$$

Q. What is wave function? Derive one dimensional Schrodinger time dependent equation and convert it in to time independent form?

Wave function ψ (x, t):-

Wave is oscillatory disturbance travelling through the medium. For every wave there are some physical quantity that varies with position x and time t. For electromagnetic wave, there is variation in electric field and magnetic field, for sound wave there is variation in pressure. De 'Broglie's wave associated with moving particle is represented by function ψ, that varies with position x and time t. Function ψ (x,t) is complex and square of this function gives the probability of locating the particle.

Schrodinger time dependent equation:



Consider a particle of mass 'm' moving with velocity 'v' in a force of field i.e force is acting on the particle.

The total energy of the particle is

E = K.E. + P.E.

$$E = \frac{p^2}{2m} + V \text{ --------------- (1)}$$

The De 'Broglie's wave associated with moving particle is given by function ψ (x.t).

The function ψ (x.t) associated with the particle may be a sine, cosine or exponential function of $(\omega t - kx)$

It is given by ψ (x.t) = A $e^{-i(\omega t - kx)}$

Multiply equation (1) by ψ (x.t)

$$E\,\psi\,(\mathrm{x.t}) = \frac{p^2\psi\,(\mathrm{x.t})}{2m} + V\,\psi\,(\mathrm{x.t}) \text{ ------------------ (2)}$$

To determine $E\,\psi\,(\mathrm{x.t})$ and $p^2\psi\,(\mathrm{x.t})$

ψ (x.t) = A $e^{-i(\omega t - kx)}$

$$\omega = 2\pi\gamma \text{ and } E = h\gamma$$

$$\omega = \frac{2\pi E}{h} = \frac{E}{\hbar}$$

And $K = \frac{2\pi}{\lambda}$ and λ = $\frac{h}{p}$

$$k = \frac{2\pi p}{h} = \frac{p}{\hbar}$$

Therefore ψ (x.t) = A $e^{-\frac{i}{\hbar}(Et-px)}$ ----------- (3)

To determine E ψ (x. t) differentiate eqn (3) w.r.to t

$$\frac{\partial\psi}{\partial t} = -\frac{iE\,\psi}{\hbar}$$

$$E\,\psi = i\hbar\,\frac{\partial\psi}{\partial t}$$

To determine P ψ (x. t) differentiate eqn (3) w.r.to x

$$\frac{\partial\psi}{\partial x} = \frac{i\,P\psi}{\hbar}$$

$$P\psi = \frac{\hbar}{i}\,\frac{\partial\psi}{\partial x}$$

$$P^2\psi = -\hbar^2\,\frac{\partial^2\psi}{\partial t^2}$$

Keep value of E ψ (x. t) and p^2ψ (x. t) in eqn.(2)

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V\Psi(x,t)$$

This is Schrodinger time dependent equation.

Schrodinger time independent equation:

If the motion of the particle is in free space i.e. force is not acting on the particle, then P.E. of particle is zero. We can separate the variables of function ψ (x. t)

Hence ψ (x. t) = ψ (x) Φ(t)

Therefore Schrodinger time dependent eqn. can be written as

$$i\hbar\,\psi(x)\frac{\partial\Phi(t)}{\partial t} = -\frac{\hbar^2}{2m}\Phi(t)\frac{\partial^2\psi(x)}{\partial x^2} + V\,\psi(x)\,\Phi(t)$$

Divide both side by ψ (x) Φ(t)

$$i\hbar\,\frac{1}{\Phi(t)}\frac{\partial\Phi(t)}{\partial t} = -\frac{\hbar^2}{2m}\,\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V$$

RHS of this equation is only the function of ‘t’ and LHS is only the function of ‘x’. Therefore both side equal to some constant say E

$$-\frac{\hbar^2}{2m}\,\frac{1}{\psi(x)}\frac{\partial^2\psi(x)}{\partial x^2} + V = E$$

Multiply both sides by ψ (x)

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+V\psi(x)=E\psi(x)$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+(E-V)\psi(x)=0$$

This is Schrodinger time independent equation.

Application of Schrodinger time independent equation:-

1) **Motion of particle in a free space:-**

Consider a particle of mass ‘m’ moving with velocity ‘v’ in a free space i.e. particle is not acted upon by any force. Therefore P.E. V = 0

The Schrodinger time independent equation is

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+(E-V)\psi(x)=0$$

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2}+E\psi(x)=0$$

$$\frac{\partial^2\psi(x)}{\partial x^2}+\frac{2m}{\hbar^2}E\psi(x)=0$$

$$\frac{\partial^2\psi(x)}{\partial x^2}+K^2\psi(x)=0$$

$$Where\ K^2=\frac{2mE}{\hbar^2}$$

But $E=\frac{p^2}{2m}$

Therefore K= $\frac{1}{\lambda}$

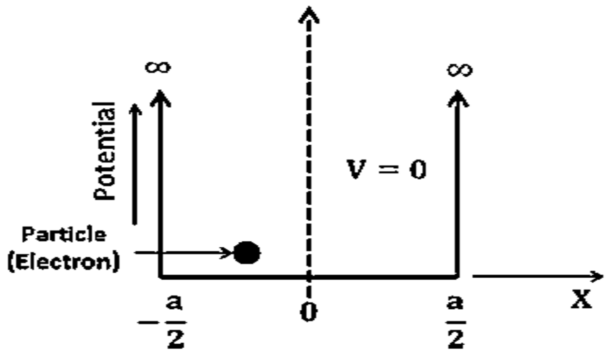
Thus K describes the wave properties of particle.

$$E=\frac{K^2\hbar^2}{2m}$$

Thus energy values are continuous.

Q. Show that energy of an electron in the box varied as the square of natural numbers?

2) Motion of particle in a box of infinite potential well:-



Consider a particle of mass 'm' moving with velocity 'v' in a box of length 'a' and infinite height. Potential energy of the particle is constant with in a box. Assume P.E. V = 0 with in a box. P.E. V= ∞ at the walls and outside the box.

The Schrodinger time independent equation is

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi\,(x)}{\partial x^2} + (E - V)\psi\,(x) = 0$$

Since within box V = 0

$$\frac{\hbar^2}{2m}\frac{\partial^2\psi\,(x)}{\partial x^2} + E\psi\,(x) = 0$$

$$\frac{\partial^2\psi\,(x)}{\partial x^2} + \frac{2m}{\hbar^2}E\psi\,(x) = 0$$

$$\frac{\partial^2\psi\,(x)}{\partial x^2} + K^2\psi\,(x) = 0 \quad \text{------------ (1)}$$

$$Where\ K^2 = \frac{2mE}{\hbar^2}$$

Solution of eqn (1) is $\psi = A\cos Kx + B\sin Kx$

The boundary conditions are at x = 0 and x = a , $\psi = 0$

B sin Ka = 0

Ka = nπ

$$K = \frac{n\pi}{a}$$

But $E = \frac{K^2\hbar^2}{2m}$

$$E = \frac{n^2\hbar^2\pi^2}{2ma^2} = \frac{n^2h^2}{8ma^2}$$ where n = 1,2,3,4

Thus energy values of the particle are discrete.

Thus energy of an electron in the box varied as the square of natural numbers.

An electron is bound in a one dimensional potential well of width 2 A^0 and of infinite height. Find its energy values in ground state and first two excited states.

Given : $m = 9.1 \times 10^{-31}$ kg,

$a = 2 \times 10^{-10}$m

$h = 6.63 \times 10^{-34}$ J – s,

$E_0, E_1, E_2 = ?$

We know, $E = \frac{n^2h^2}{8\,m\,a^2}$

For ground state, $n = 1, E = E_0$

$\therefore E_0 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 1.5 \times 10^{-18}$J

For first excited state, $n = 2, E = E_1$

$\therefore E_1 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 6 \times 10^{-18}$J

For second excited state, $n = 3, E = E_2$

$\therefore E_2 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{3^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 1.35 \times 10^{-17}$J

An electron is bound in a one dimensional potential well of width 2 A^0 and of infinite height. Find its energy values in ground state and first two excited states.

Given : $m = 9.1 \times 10^{-31}$ kg,

$a = 2 \times 10^{-10}$m

$h = 6.63 \times 10^{-34}$ J – s,

$E_0, E_1, E_2 = ?$

We know, $E = \frac{n^2h^2}{8\,m\,a^2}$

For ground state, $n = 1, E = E_0$

$\therefore E_0 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 1.5 \times 10^{-18}$J

For first excited state, $n = 2, E = E_1$

$\therefore E_1 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 6 \times 10^{-18}$J

For second excited state, $n = 3, E = E_2$

$\therefore E_2 = \frac{n^2h^2}{8\,m\,a^2}$

$= \frac{3^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2 \times 10^{-10})^2}$

$= 1.35 \times 10^{-17}$J

An electron is trapped in a one dimensional box of length 0.1 nm. Calculate the energy required to excite the electron from its ground state to the 4th excited state.

Given : $m = 9.1 \times 10^{-31}$ kg,

$a = 0.1 \times 10^{-9}$ m,

$h = 6.63 \times 10^{-34}$ J − s,

$E_4 - E_0 = ?$

$$E = \frac{n^2h^2}{8\,m\,a^2}$$

For ground state, $n = 1, E = E_0$

$$\therefore E_0 = \frac{n^2h^2}{8\,m\,a^2}$$

$$= \frac{1^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$= 6.04 \times 10^{-18} \text{J}$$

For fourth excited state, $n = 5, E = E_4$

$$\therefore E_4 = \frac{n^2h^2}{8\,m\,a^2}$$

$$= \frac{5^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (0.1 \times 10^{-9})^2}$$

$$= 1.51 \times 10^{-16} \text{J}$$

$$E_4 - E_0 = 1.51 \times 10^{-16} - 6.04 \times 10^{-18}$$

$$= 1.45 \times 10^{-16} \text{J}$$

$$= \frac{1.45 \times 10^{-16}}{1.6 \times 10^{-19}} = 906.25 \text{ eV}$$

An electron is bound by a potential which closely approaches an infinite square well of width 2.5 x 10^{-10} m. Calculate the first lowest permissible energy for the electron.

Given : $m = 9.1 \times 10^{-31}$ kg,

$a = 2.5 \times 10^{-10}$ m $\qquad h = 6.63 \times 10^{-34}$ J − s

For lowest permissible Energy level, $E = E_1$ and $n = 2$

$$E = \frac{n^2h^2}{8\,m\,a^2}$$

For lowest permissible Energy level, $n = 2, E = E_1$

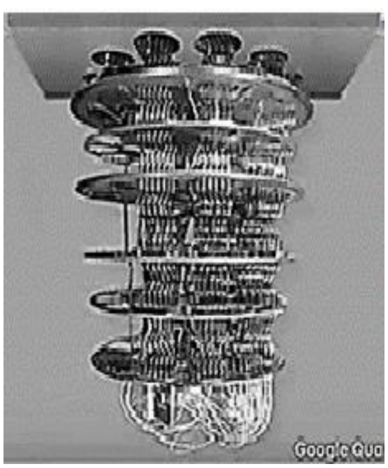
$$\therefore E_1 = \frac{n^2h^2}{8\,m\,a^2} = \frac{2^2 \times (6.63 \times 10^{-34})^2}{8 \times 9.1 \times 10^{-31} \times (2.5 \times 10^{-10})^2} = 3.864 \times 10^{-18} \text{J}$$

$$= 24.15 \text{ eV}$$

Q. Write short note on quantum computing?

Quantum computing:

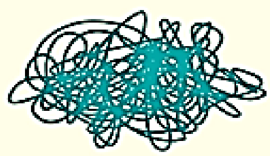
1) Quantum computing means developing computer technology based on the principles of quantum theory. Quantum computers can use certain phenomena from quantum mechanics, such as superposition and entanglement, to represent and process data.
2) **Quantum superposition** means any two or more quantum states are added together and result will be another valid state.
3) **Quantum entanglement** means two or more particles are linked in such a way that it is impossible for them to be described independently even if separated by a large distance.
4) Classical and quantum computers both store data and process data as binary code 0 and 1. Quantum computers encode information as quantum bits or qubits, which can exist in superposition.
5) Qubits represent atoms, ions, photons or electrons and their respective control devices that are working together to act as computer memory and a processor.
6) Computing capability of quantum computer is far greater than supercomputer. Quantum superposition and entanglement create an enormously enhanced computing power. In an ordinary computer two bits 0 and 1, can store only one of four binary configurations (00, 01, 10, or 11) at any given time while in a quantum computer two qubit can store all four numbers simultaneously, because each qubit represents two values. If more qubits are added, the increased capacity is expanded exponentially.



Quantum computer

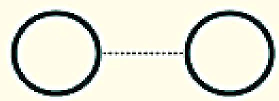
Classical computer

Point of comparison	***Classical Computing***	***Quantum Computing***
Information representation	A bit: either 0 or 1	A qubit: a superposition of 1 and 0
Number of simultaneous calculations	1	Multiple
Method of calculation	Moving bits through logic gates	Altering states of atoms
Information delivered	Information can be copied without being disturbed	Information cannot be copied or read without being disturbed
Information behavior	One single direction	Spread-out to many routes simultaneously like overlapping waves
Noise tolerance	High: Information can be carried in a noisy channel	Low: The delivering channel needs to be noiseless
Security	Lower: Eavesdropper can break into the communication with high computing power	Higher: Any interruption of communication will be detected by communicating parties
Computation/ Communication cost	Higher as computing or communication volume increases	Lower as computing or communication volume increases



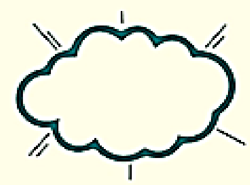
SUPERPOSITION

Superposition describes a particle's ability to exist across many possible states at the same time. So the state of a particle is best described as a "superposition" of all those possible states.



ENTANGLEMENT

Quantum entanglement refers to a situation in which two or more particles are linked in such a way that it is impossible for them to be described independently even if separated by a large distance.



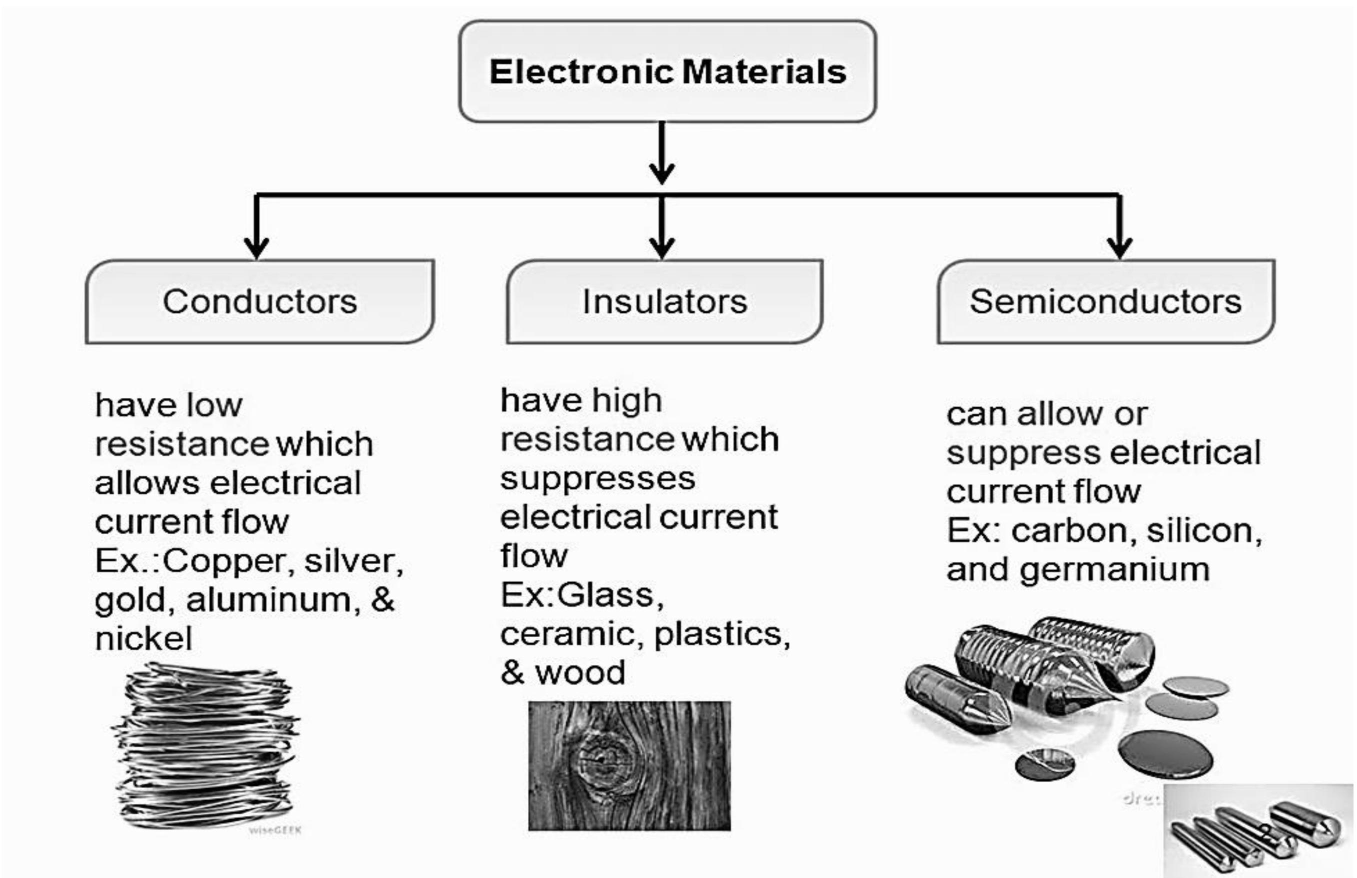
OBSERVATION

Superposition and entanglement only exist as long as quantum particles are not observed or measured. "Observing" the quantum state yields information but results in the collapse of the system.

SEMICONDUCTOR PHYSICS

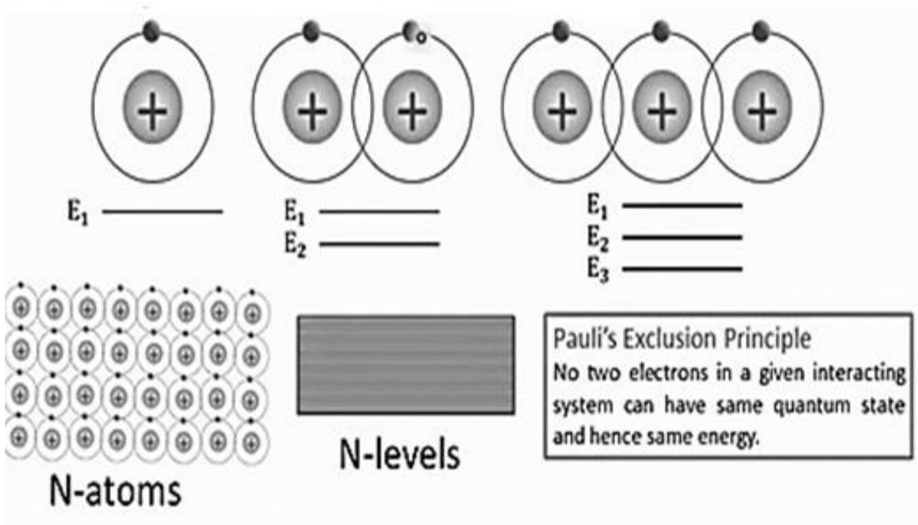
(Prerequisites: Intrinsic and extrinsic semiconductors, Energy bands in conductors, semiconductors and insulators, Semiconductor diode, I-V characteristics in forward and reverse bias)

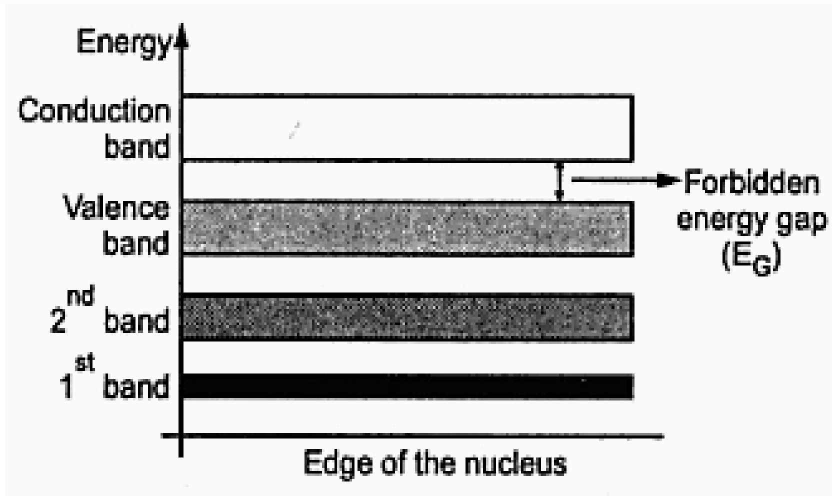
Direct & indirect band gap semiconductor; Fermi level; Fermi dirac distribution; Fermi energy level in intrinsic & extrinsic semiconductors; effect of impurity concentration and temperature on fermi level; mobility, current density;Hall Effect; Fermi Level diagram for p-n junction (unbiased, forward bias, reverse bias); Applications of semiconductors: LED, Zener diode, Photovoltaic cell.



CLASSIFICATION OF SOLIDS:

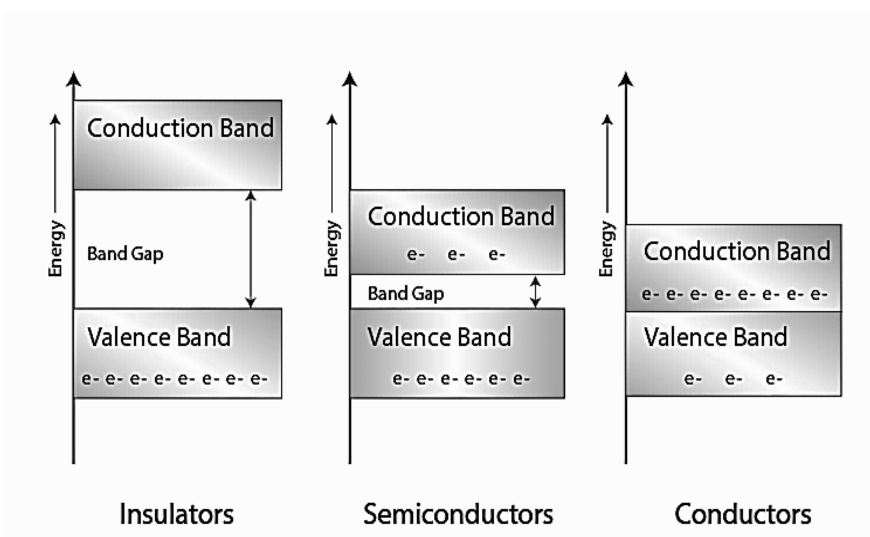
Formation Energy Band in solid:





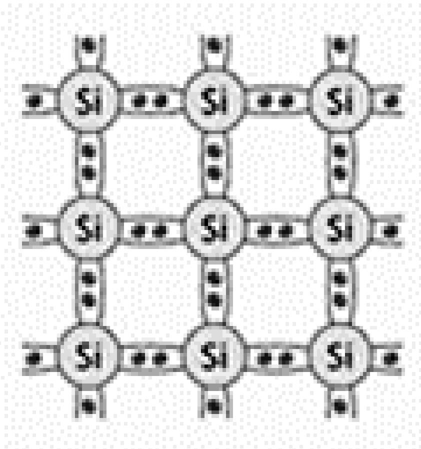
Energy band diagram

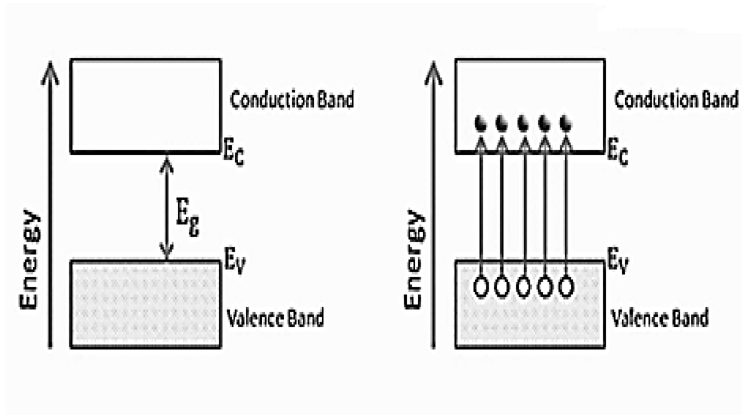
Depending upon forbidden energy gap, solid materials are classified as



INTRINSIC & EXTRINSIC SEMICONDUCTOR:

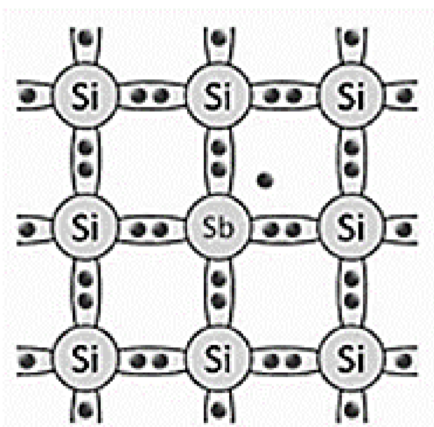
Intrinsic Semiconductor:

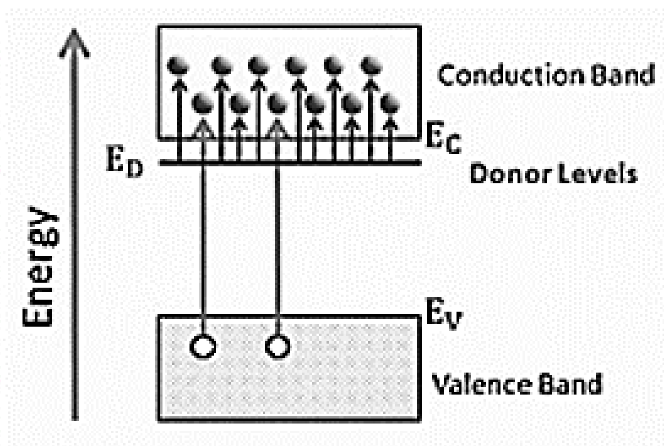




Extrinsic Semiconductor (Doped semiconductor):

N-type semiconductor:





Direct and Indirect band gap semiconductor:

1) The solid crystals are formed when the isolated atoms are brought together. However, when two atoms are brought close to each other, it leads to intermixing of electrons in the valence shell. As a result, the number of permissible energy levels is formed, which is called an energy band.

2) Each band is formed due to the splitting of one or more atomic energy levels. Therefore, the minimum number of states in a band equals twice the number of atoms in the material. The reason for the factor of two is that every energy level can contain two electrons with opposite spin.

3) Band gap is the difference in energy between the valence band and the conduction band of a solid material that consists of the range of energy values forbidden to electrons in the material.

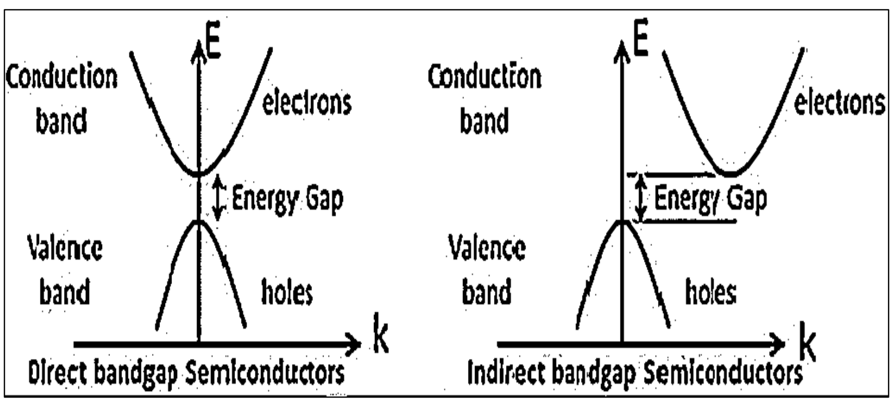
4) In semiconductor physics, the band gap of a semiconductor can be of two basic types, a direct band gap or an indirect band gap. Therefore another way of classifying semiconductors based on their band structure is

   a) Direct band gap semiconductor b) Indirect band gap semiconductor

5) The difference between direct and indirect band gap semiconductors is related to their band structure. Electrons in solids have a wave like character. An electron wave is characterized by a wave vector k. Thus, for crystalline materials it possible to plot E vs. vector k diagrams. These are related to the simple band diagrams that show the valence and conduction band.

6) The band gap is called "direct" if the crystal momentum (Vector k) of electrons and holes is the same in both the conduction band and the valence band. In direct band-gap (DBG) semiconductor the maximum energy level of the valence band aligns with the minimum energy level of the conduction band with respect to momentum.

7) The band gap is called "indirect" if the crystal momentum (Vector k) of electrons and holes is not same in both the conduction band and the valence band. In indirect band-gap (IBG) semiconductor the maximum energy level of the valence band misaligned with the minimum energy level of the conduction band with respect to momentum.



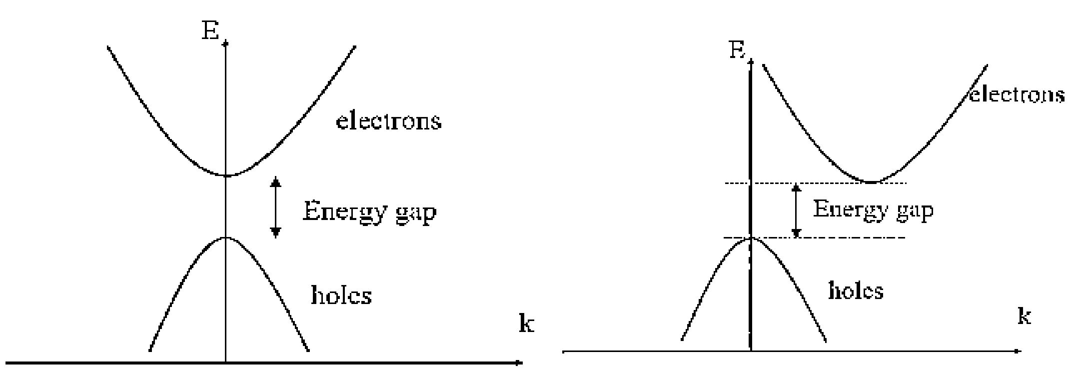
8) Electrons from the valence band can be excited to the conduction band by either thermal excitation or by optical absorption. When the electron returns to the valence band the energy is released either as heat or as photons.

9) An electron can directly emit a photon. In an "indirect" gap, a photon cannot be emitted because the electron must pass through an intermediate state and transfer momentum to the crystal lattice.

10) An example of direct band gap material includes some III-V materials such as InAs, GaAs are also known as compound semiconductor. Indirect band gap materials include Si, Ge are also known as elemental semiconductor.

Q. Differentiate between direct and indirect band gap semiconductors:

Direct band-gap (DBG) semiconductor	Indirect band-gap (IBG) semiconductor
1) A direct band-gap (DBG) semiconductor is one in which the maximum energy level of the valence band aligns with the minimum energy level of the conduction band with respect to momentum.	1) An Indirect band-gap (IBG) semiconductor is one in which the maximum energy level of the valence band and the minimum energy level of the conduction band are misaligned with respect to momentum.
2) In a DBG semiconductor, a direct recombination takes place with the release of the energy in the form of photon of energy equal to the energy difference between the recombining particles.	2) In case of a IBG semiconductor, due to a relative difference in the momentum, first, the momentum is conserved by release of energy and only after the both the momenta align themselves, a recombination occurs accompanied with the release of energy in the form of phonon (heat).
3) The probability of a radiative recombination is high.	3) The probability of a radiative recombination is comparatively low.
4) The efficiency factor of a DBG semiconductor is higher. Thus, DBG semiconductors are always preferred over IBG for making optical sources.	4) The efficiency factor of an IBG semiconductor is lower.
5) Example, Gallium Arsenide (GaAs).	5) Example, Silicon and Germanium



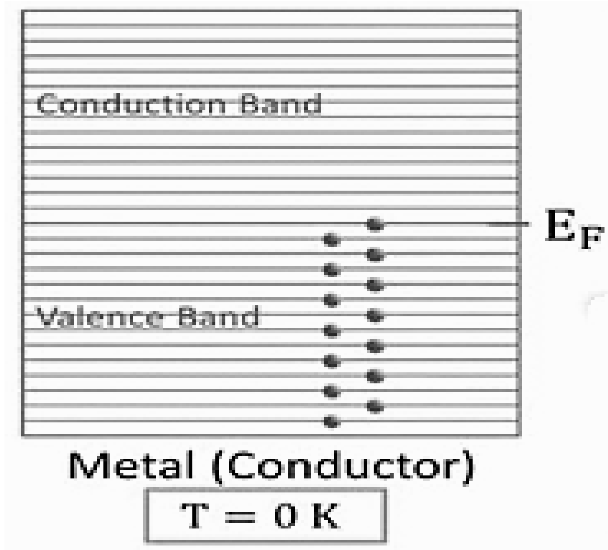
Direct bandgap semiconductors

Indirect bandgap semiconductors

k=momentum

FERMI ENERGY LEVEL IN CONDUCTORS (Metal):

Q. What is Fermi level in metal? Write Fermi-Dirac distribution function and explain the term used in it?



1) In conductor's valence and conduction bands are overlap on each other. Therefore continuous range of energy levels is available for the electrons.

2) The electrons are filling up in to the various energy levels such that first lowest energy level is filled by the pair of electrons and then second lowest energy level and so on till all the electrons occupy the particular energy level. The upper most energy level occupied by the electrons in metal is known as Fermi level and its energy is known as Fermi energy. Thus Fermi level is allowed energy level in conductors.

3) The distribution of electrons of energy E in various energy levels is given by Fermi-Dirac distribution function,

$$f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

f(E) is the probability of occupancy for energy level E

E_F is Fermi energy

T is temperature in ^{0}K and

$k = 1.38 \times 10^{-23}\,J/K = 8.625 \times 10^{-5}\,eV/K$

Fermi function F(E) gives the probability that an electrons of energy E occupying particular energy level.

Q. Explain the variation of Fermi level with temperature in metal?

4) Variation of Fermi energy level with temp. in metal:-

At T = 0^0K,

At $T = 0$ K, For $E > E_F$

$$e^{\left(\frac{E-E_F}{kT}\right)} = e^{\frac{+ve\ number}{0}} = e^{+\infty} = \infty$$

$$\therefore f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+\infty} = 0$$

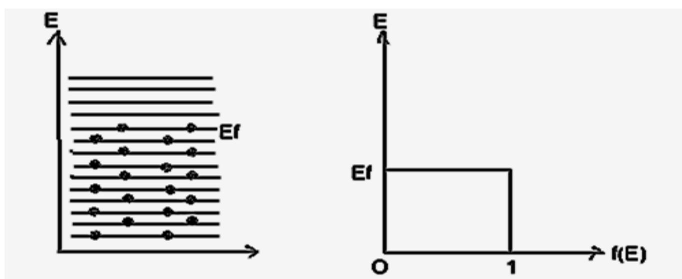
When E >Ef, F(E) = 0 means all the levels above Ef are vacant.

At $T = 0$ K, For $E < E_F$

$$e^{\left(\frac{E-E_F}{kT}\right)} = e^{\frac{-ve\ number}{0}} = e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$$

$$\therefore f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+0} = 1$$

When E <Ef, F(E) = 1 means all the levels below Ef are filled with electrons.

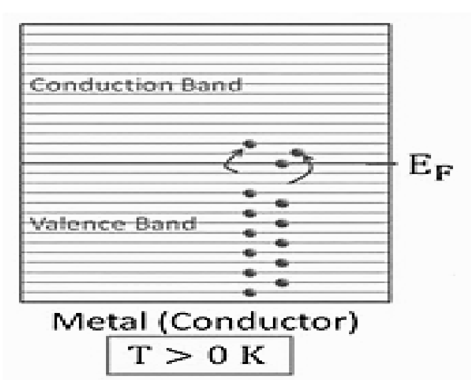


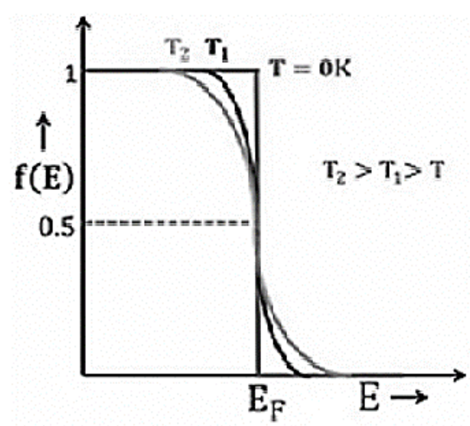
- **At T > O^0K temp.**

Some electrons near to Ef are excited to energy levels above Ef.

When E<Ef, function F(E) slightly less than 1, i.e probability of electrons below Ef decreases.

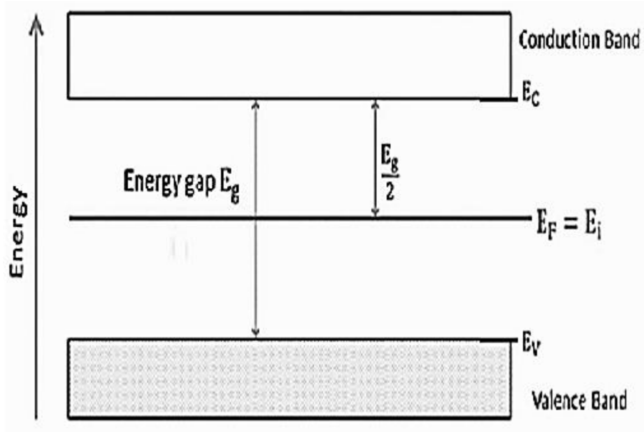
When E >Ef, function F(E) slightly greater than 1, i.e probability of electrons above Ef increases.





Q. Show that in intrinsic semiconductor, Fermi level always lies at the middle between the valence and conduction band?

FERMI LEVEL IN INTRINSIC SEMICONDUCTOR:-



1) In semiconductors at temp. T = O^0K, the valance band is completely filled by the electrons and conduction band is empty. At temp. T^0K, some electrons from the valence band can jump in to the conduction band, leaving behind equal number of holes in the valence band. Thus in semiconductors the charged carriers are electrons and holes.

2) At temp. T^0K, the density of free electrons in the conduction band and density of holes in the valence band is equal. Therefore Fermi level always at the middle between the valence band and conduction band.

3) Thus in semiconductor Fermi level is not an allowed energy level for electrons in the semiconductor, but it is reference level between the valence band and conduction band.

If E_v – energy of upper level of valence band.
E_c – energy of bottom level of conduction band.
E_f – Fermi energy level.
E_g– energy gap between valence band and conduction band.

Therefore, $Ef = \frac{Ec+Ev}{2} = \frac{Eg}{2}$

Consider at temp. T^0K,

n_c – Concentration of free electrons in conduction band.
n_v– Concentration of holes in the valence band.
N_c– Effective density of states in the conduction band.
N_v – Effective density of states in the valence band.

$$n_c = Nc\ e^{-(Ec - Ef)/KT}$$

$$n_v = Nv\ e^{-(Ef - Ev)/KT}$$

$$\text{but } n_c = n_v$$

$$N_c\, e^{-(E_c - E_f)/KT} = N_v\, e^{-(E_f - E_v)/KT}$$

$$e^{-(E_c - E_f)/KT} = \frac{N_v}{N_c}\, e^{-(E_f - E_v)/KT}$$

But Nv = Nc, therefore Nv / Nc =1

$$e^{-(E_c - E_f)/KT} = e^{-(E_f - E_v)/KT}$$

Take logarithm on both sides,

$$\frac{-(E_c - E_f)}{KT} = \frac{-(E_f - E_v)}{KT}$$

$$-E_c + E_f + E_f - E_v = 0$$

$$\text{Therefore, } \boldsymbol{Ef} = \frac{\boldsymbol{Ec+Ev}}{\boldsymbol{2}} = \frac{\boldsymbol{Eg}}{\boldsymbol{2}}$$

Thus in intrinsic semiconductor, Fermi level always at the middle between the valence band and conduction band.

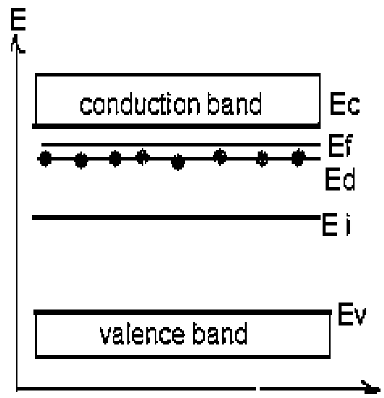
FERMI LEVEL IN EXTRINSIC SEMICONDUCTOR:-

Q. Explain the variation of Fermi level with temperature & impurity concentration in N-type semiconductor?

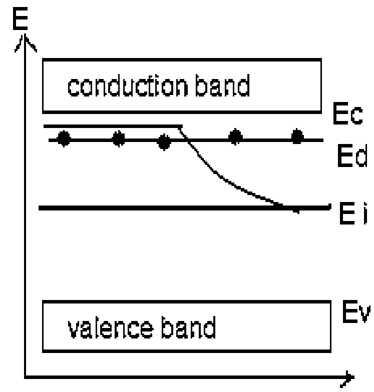
In extrinsic semiconductor, density of free charge carriers (electrons or holes) in the conduction band and valence band increases with temperature and impurity concentration, hence Fermi level depends upon the temp. and impurity concentration.

1) **Variation of Ef with temperature**:-

In extrinsic semiconductor the Fermi level lies in upper half or lower half of the energy gap depending upon the concentration of majority charge carriers.



Ef in N- type semiconductor



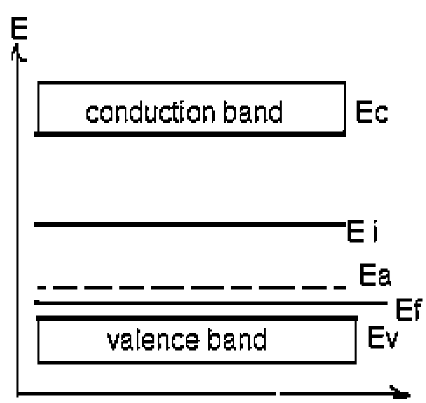
variation of Ef with temp.

In N – type semiconductor, the density of free electrons in the conduction band is greater than density of holes in the valence band. The Fermi level lies at the middle between the bottom level of conduction band Ec and donor level of impurity atoms Ed.

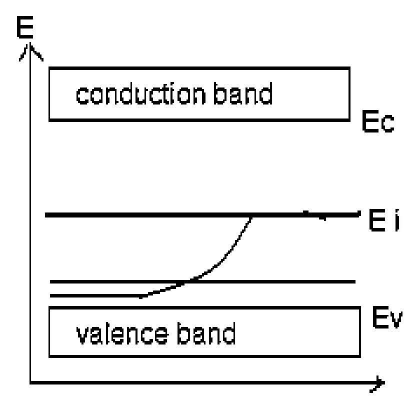
Therefore, $$E_f = \frac{E_c + E_d}{2}$$

As the temp. Increases, the donor level are depleted by the electrons from the valence band and density of holes in the valence band increases. Therefore Fermi level Ef go on decreasing. At particular temp. the density of free electrons in the conduction band becomes equal to the density of holes in the valence band and Fermi level reaches to intrinsic level Ei. The material loses its extrinsic nature and become intrinsic semiconductor. The Fermi level Ef becomes independent of temp.

In P – type semiconductor, the Fermi level lies at the middle between the top level of valence band Ev and acceptor level of impurity atoms E_A.



Ef in P-type semiconductor

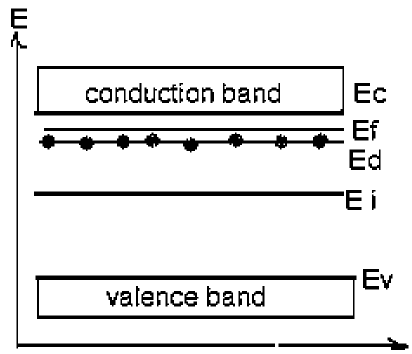


variation of Ef with temp.

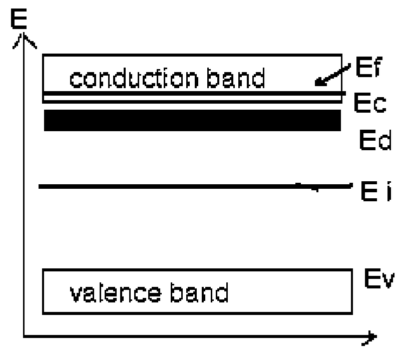
Therefore, $$E_f = \frac{E_c + E_A}{2}$$

As the temp. Increases, the Fermi level go on increasing. At particular temp., the Fermi level Ef reaches to intrinsic level Ei and material loses its extrinsic nature and become intrinsic semiconductor.

2) **Variation of Ef with impurity concentration**:



Ef in N- type semiconductor



variation of Ef with impurity concentration

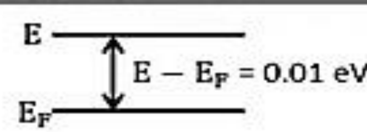
In N – type semiconductor at low impurity concentration, the donor atoms are isolated from each other and therefore single donor energy level Ed is formed below the conduction band. The Fermi level lies at the middle between the bottom level of conduction band Ec and donor level of impurity atoms Ed.

$$\text{Therefore, } E_f = \frac{E_c + E_d}{2}$$

As the impurity concentration increases, the number of donor atoms in the donor level increases. These atoms are interacting with each other, therefore donor level splits in to no. of discrete energy levels and donor energy band is formed. The width of donor energy band increases with increase in impurity concentration. The Fermi level Ef shift close to conduction band and at higher concentration Ef may enter in to the conduction band.

Calculate probability of non-occupancy for the energy level which lies 0.01 eV above the Fermi energy level at 27 ^{0}C.

Given : $T = 27\,^0C = 300\,^0K$, $k = 8.625 \times 10^{-5} \frac{eV}{^0K}$



$$\text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\therefore \text{Probability of non-occupancy} = 1 - f(E) = 1 - \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$= 1 - \frac{1}{1+e^{\left(\frac{0.01}{8.625\times10^{-5}\times300}\right)}}$$

$$= 0.595$$

Fermi level for silver is 5.5 eV. Find out the energy for which the probability of occupancy at 300 K is 0.9.

Given : $T = 27\,°C = 300^0K$, $k = 8.625 \times 10^{-5}\,\frac{eV}{°K}$, $E_F = 5.5\text{ eV}$

when $f(E) = 0.9, E = ?$

$$\textbf{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\therefore 1 + e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)}$$

$$\therefore e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)} - 1$$

$$\therefore \frac{E-E_F}{kT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore E - E_F = kT\ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore E = E_F + kT\ln\left(\frac{1}{f(E)} - 1\right)$$

For $f(E) = 0.9$

$$E = 5.5 + 8.625 \times 10^{-5} \times 300\ln\left(\frac{1}{0.9} - 1\right)$$

$$= 5.443\text{ eV}$$

Fermi level in potassium is 2.1 eV. What are the energies for which the probability of occupancy at 300 K are 0.99 and 0.01.

Given : $T = 27\,°C = 300^0K$, $k = 8.625 \times 10^{-5}\,\frac{eV}{°K}$, $E_F = 2.1\text{ eV}$

when $f(E) = 0.99$ and $f(E) = 0.01, E = ?$

$$\textbf{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\therefore 1 + e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)}$$

$$\therefore e^{\left(\frac{E-E_F}{kT}\right)} = \frac{1}{f(E)} - 1$$

$$\therefore \frac{E-E_F}{kT} = \ln\left(\frac{1}{f(E)} - 1\right)$$

$$\therefore E = E_F + kT\ln\left(\frac{1}{f(E)} - 1\right)$$

For $f(E) = 0.99$

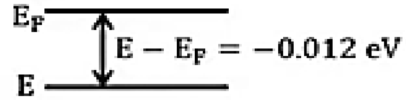
$$E = 2.1 + 8.625 \times 10^{-5} \times 300\ln\left(\frac{1}{0.99} - 1\right) = 1.981\text{ eV}$$

For $f(E) = 0.01$

$$E = 2.1 + 8.625 \times 10^{-5} \times 300\ln\left(\frac{1}{0.01} - 1\right) = 2.218\text{ eV}$$

In a solid, consider the energy level lying 0.012 eV below the Fermi energy level at 27 ^{0}C. What is the probability of this level being occupied by an electron?

Given : $T = 27\,^0C = 300\,^0K$, $k = 8.625 \times 10^{-5}\,\frac{eV}{°K}$



$$\text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}}$$

$$\therefore \text{Probability of occupancy } f(E) = \frac{1}{1+e^{\left(\frac{E-E_F}{kT}\right)}} = \frac{1}{1+e^{\left(\frac{-0.012}{8.625\times10^{-5}\times300}\right)}}$$

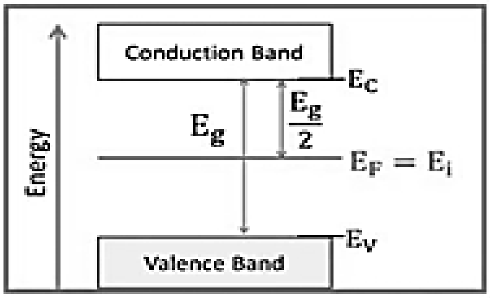
$$= 0.614$$

What is the probability of an electron being thermally excited to conduction band in intrinsic Si at 27 ^{0}C. The band gap energy of Si is 1.12 eV.

Given : T = 27 ^{0}C = 300 ^{0}K, $k = 8.625 \times 10^{-5}\ \frac{eV}{^0K}$, $E_g = 1.12$ eV

For intrinsic semiconductor $E_F = \frac{E_C + E_V}{2}$

$$\therefore E_C - E_F = \frac{E_g}{2} = \frac{1.12}{2} = 0.56 \text{ eV}$$



$$\therefore \text{Probability of occupancy} = f(E_C) = \frac{1}{1 + e^{\left(\frac{E_C - E_F}{kT}\right)}}$$

$$= \frac{1}{1 + e^{\left(\frac{0.56}{8.625 \times 10^{-5} \times 300}\right)}}$$

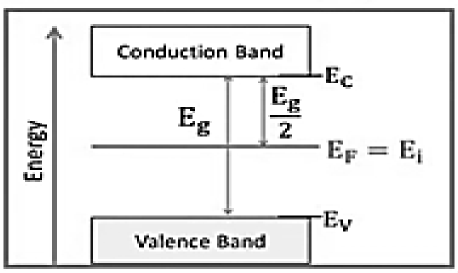
$$= 3.99 \times 10^{-10}$$

What is the probability of an electron being thermally promoted to conduction band in diamond at 27^{0}C if band gap is 5.6 eV wide.

Given : T = 27 ^{0}C = 300 ^{0}K, $k = 8.625 \times 10^{-5}\ \frac{eV}{^0K}$, $E_g = 5.6$ eV

For diamond, $E_F = \frac{E_C + E_V}{2}$

$$\therefore E_C - E_F = \frac{E_g}{2} = \frac{5.6}{2} = 2.8 \text{ eV}$$



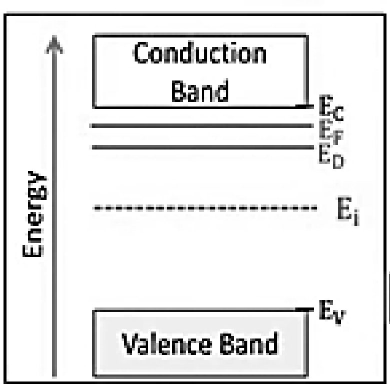
$$\therefore \text{Probability of occupancy} = f(E_C) = \frac{1}{1 + e^{\left(\frac{E_C - E_F}{kT}\right)}}$$

$$= \frac{1}{1 + e^{\left(\frac{2.8}{8.625 \times 10^{-5} \times 300}\right)}}$$

$$= 1.09 \times 10^{-47}$$

In an n-type semiconductor, the Fermi level lies 0.4 eV below the conduction band. If the concentration of donor atoms is doubled, find the new position of the Fermi level w.r.t. conduction band.

Given : $E_C - E_F = 0.4$ eV



Temperature is not mentioned.
Let us assume T = 300 K

Electron concentration is given by

$$n = N_C e^{-(E_C - E_F)/kT} \quad (1)$$

When donor concentration is doubled, electron concentration will also get doubled and fermi level will be shifted to E_{F2}

$$2n = N_C e^{-(E_C - E_{F2})/kT} \quad (2)$$

Divide (2) by (1)

$$\frac{e^{-(E_C - E_{F2})/kT}}{e^{-(E_C - E_F)/kT}} = 2$$

$$\therefore e^{\frac{-(E_C - E_{F2}) + (E_C - E_F)}{kT}} = 2$$

$$\therefore -(E_C - E_{F2}) + (E_C - E_F) = kT \ln(2)$$

$$\therefore (E_C - E_{F2}) = (E_C - E_F) - kT \ln(2)$$

$$\therefore (E_C - E_{F2}) = 0.4 - 8.625 \times 10^{-5} \times 300 \ln(2)$$

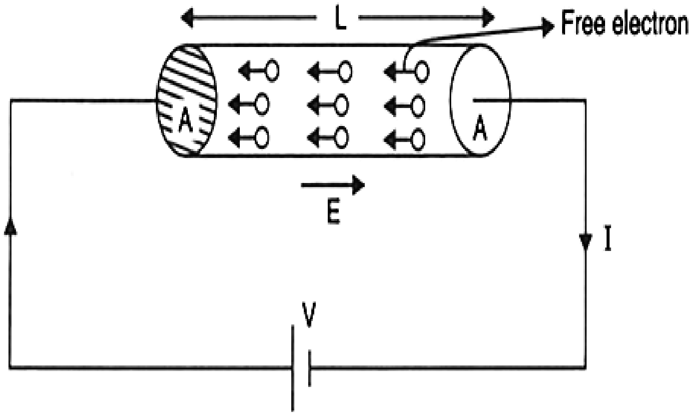
$$\therefore (E_C - E_{F2}) = 0.382 \text{ eV}$$

So, The Fermi level shifts towards the Conduction band as donor concentration increases.

Q. Write short notes on drift and diffusion current?

Q. What is mobility of charge carriers? Give its SI unit?

DRIFT CURRENT:-



In metals large numbers of free electrons are present. There is force of repulsion between these electrons. These free electrons are colliding with each other and with the atoms of the material and rebound in various directions. The motion of electrons is equivalent to Brownian motion.

When external field E is applied, the electrons are accelerated in the direction of applied field and acquire certain velocity known as driftvelocity. The applied electric field does not stop the collision of electrons with the atoms and renounce in various directions, but the electrons are drifted in the direction of applied field and constitute an electric current. This current is known as drift current. The drift velocity per unit electric field is known as mobility of charge carriers. The SI unit of mobility is m^2 / volt.Second. The mobility of electrons is greater than holes.

If n – density of free electrons

e - electron charge.

Ve – drift velocity of electrons.

A – area of cross section

Therefore drift current, $I = n\ e\ V_e\ A$

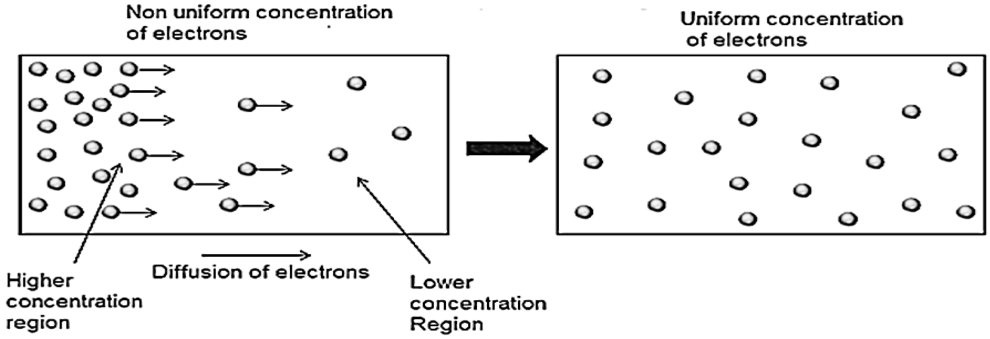
If E is applied electric field, the mobility of electrons $\mu_e = V_e / E$

$V_e = \mu_e E$

$I = n\ e\ \mu_e E\ A$

Drift current density, $\mathbf{J_{n(drift)} = I / A = n\ e\ \mu_e E}$

DIFFUSION CURRENT:-



In semiconductors, the charge carriers are electrons and holes. The concentration of charge carriers may not be same throughout the material. The change in concentration of charge carriers per unit length is called concentration gradient (dn / dx). The charge carriers which are concentrate in some region having same charge and therefore force of repulsion between them. There is tendency of charge carriers to diffuse from region of higher concentration to the region of lower concentration till all the charge carriers are distributed uniformly throughout the material. This motion of charge carriers constitute an electric current known as diffusion current.

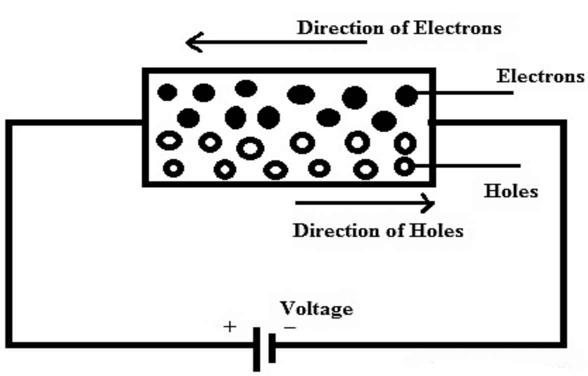
The diffusion current density is directly proportional to the magnitude of charge and concentration gradient.

$$J_{n(diffusion)} \propto (-e) (- dn / dx)$$

$$\boxed{J_{n(diffusion)} = Dn\ e\ dn / dx}$$

Where Dn – diffusion constant for electrons.

CURRENT CONDUCTION IN SEMICONDUCTOR:-



Consider a piece of intrinsic semiconductor of length l and area of cross section A. If V is the P.D. applied across the semiconductor, then current flows through the semiconductor is due to motion of electrons and holes in opposite direction.

The current flows though the external circuit is due to electrons only. Therefore total current flowing through the semiconductor is

$$I = Ie + Ih \text{ ------------------- } (1)$$

If n – density of free electrons

e - electron charge.

Ve – drift velocity of electrons.

A – area of cross section

Therefore drift current, I = n e Ve A

If E is applied electric field, **the mobility of electrons** **$\mu e = Ve / E$**

$Ve = \mu_e E$

$Ie = n\, e\, \mu_e E\, A$

Similarly, $Ih = p\, e\, \mu_h\, E\, A$

Therefore, eqn.(1)=> $I = n\, e\, \mu_e E\, A + p\, e\, \mu h\, E\, A$

But in intrinsic semiconductor n = p = ni intrinsic carrier density

$I = nie(\mu e + \mu h)\, E\, A$

Current density **$J = I / A = ni\, e(\mu e + \mu h)\, E$**

But $E = V / l$

$I = nie(\mu e + \mu h)\, (V / l)\, A$

$V / I = (1 / ni\, e(\mu e + \mu h)\;\; l / A$

$R = \rho\, (l / A)$

$\rho = 1 / nie(\mu_e + \mu_h)$ resistivity of semiconductor.

Reciprocal of resistivity is conductivity, **$\sigma = 1 / \rho = nie(\mu_e + \mu_h)$**

Find resistivity of Ge at 300 ^{0}K. Given density of carriers is 2.5 x 10^{19} / m^3. Mobility of electrons is 0.39 m^2/V-sec, mobility of holes = 0.19 m^2/V-sec.

Given : $n_i = 2.5 \times 10^{19} / m^3$,

$\mu_e = 0.39\ m^2/V\text{-sec}$, $\mu_h = 0.19\ m^2/V\text{-sec}$

For intrinsic semiconductor, conductivity is given by –

$$\sigma_i = n_i\, e\, (\mu_e + \mu_h)$$
$$= 2.5 \times 10^{19} \times 1.6 \times 10^{-19}\, (0.39 + 0.19)$$
$$= 2.32\ (\Omega - m)^{-1}$$

$$\text{Resistivity} = \rho = \frac{1}{\sigma} = \frac{1}{2.32} = 0.43\ \Omega - m$$

The resistivity of intrinsic InSb at room temperature is 2×10^{-4} ohm-cm. If the mobility of electron is 6 m^2/V-s and mobility of hole is 0.2 m^2/V-s, calculate its intrinsic carrier density.

Given : Resistivity $\rho = 2 \times 10^{-4}$ ohm − cm = 2×10^{-6} ohm − m

$\mu_e = 6 \frac{m^2}{Vsec}$, $\mu_h = 0.2 \frac{m^2}{Vsec}$, $n_i = ?$

For intrinsic semiconductor, conductivity is given by – $\sigma = \frac{1}{\rho} = n_i e(\mu_e + \mu_h)$

$$\therefore n_i = \frac{1}{\rho\, e\, (\mu_e + \mu_h)}$$

$$\therefore n_i = \frac{1}{2 \times 10^{-6} \times 1.6 \times 10^{-19} \times (6 + 0.2)} = 5.04 \times 10^{23}\ /m^3$$

The resistivity of intrinsic InSb at room temperature is 2×10^{-4} ohm-cm. If the mobility of electron is 6 m^2/V-s and mobility of hole is 0.2 m^2/V-s, calculate its intrinsic carrier density.

Given : Resistivity $\rho = 2 \times 10^{-4}$ ohm − cm = 2×10^{-6} ohm − m

$\mu_e = 6 \frac{m^2}{Vsec}$, $\mu_h = 0.2 \frac{m^2}{Vsec}$, $n_i = ?$

For intrinsic semiconductor, conductivity is given by – $\sigma = \frac{1}{\rho} = n_i e(\mu_e + \mu_h)$

$$\therefore n_i = \frac{1}{\rho\, e\, (\mu_e + \mu_h)}$$

$$\therefore n_i = \frac{1}{2 \times 10^{-6} \times 1.6 \times 10^{-19} \times (6 + 0.2)} = 5.04 \times 10^{23}\ /m^3$$

Calculate the number of donor atoms which must be added to an intrinsic semiconductor to obtain the resistivity as 10^{-6} ohm-cm. Use mobility of electron = 1000 cm^2/V-sec.

Given : μ_e = 1000 cm^2/V-sec, Resistivity $\rho = 10^{-6}\ \Omega$ − cm

N_d = ?

For n-type semiconductor, conductivity is given by – $\sigma = n\, e\, \mu_e = N_d\, e\, \mu_e$

$$\therefore N_d = \frac{\sigma}{e\, \mu_e} = \frac{1}{\rho\, e\, \mu_e} = \frac{1}{10^{-6} \times 1.6 \times 10^{-19} \times 1000} = 6.25 \times 10^{21}\ /cm^3$$

6.25×10^{21} donor atoms must be added per cm^3

Find resistivity of Copper assuming that each atom contributes one free electron for conduction. Given density of Cu = 8.96 gm / cm^3, atomic weight = 63.5, Avogadro's Number = 6.023 x 10^{23} / gm-mol, Mobility of electron = 43.3 cm^2/V-sec.

Given : μ_e = 43.3 cm^2/V-sec, density ρ = 8.96 gm/cm^3, Atomic weight = 63.5

Resistivity = ?

In 63.5 gm there are 6.023×10^{23} atoms

Number of atoms per unit volume (i.e. in 8.96 gm) will be –

$$\text{Atomic density} = \frac{8.96 \times 6.023 \times 10^{23}}{63.5} = 8.4985 \times 10^{22}\ /\text{cm}^3$$

each atom contributes one free electron

$$\therefore \text{electron concentration} = n = 1 \times \text{Atomic density} = 8.4985 \times 10^{22}\ /\text{cm}^3$$

Resistivity is given by –

$$\rho = \frac{1}{\sigma} = \frac{1}{n\, e\, \mu_e} = \frac{1}{8.4985 \times 10^{22} \times 1.6 \times 10^{-19} \times 43.3} = 1.698 \times 10^{-6}\,\Omega - \text{cm}$$

Calculate conductivity of a germanium sample if donor impurity atoms are added to the extent of one part in 10^8 germanium atoms at room temperature. Assume that only one electron of each atom takes part in conduction process.
Avogadro's number = 6.023 x 10^{23} / gm-mol, Density of Ge = 5.32 gm / cm^3
Atomic weight of Ge =72.6, mobility of electrons = 3800 cm^2/volt-sec

72.6 gm Ge contains 6.023×10^{23} atoms

$$\therefore \quad 5.32\,\text{gm i.e. } 1\,\text{cm}^3 \text{ will have } \frac{6.023 \times 10^{23}}{72.6} \times 5.32 = 4.41 \times 10^{22} \text{ atoms}$$

$$\text{As donor impurity is 1 part in } 10^8 \text{, donor concentration, Nd} = \frac{4.41 \times 10^{22}}{10^8}$$

$$= 4.41 \times 10^{14}\ /\text{cm}^3$$

As only one electron of each donor atom takes part in conduction,

electron concentration = $n = 4.41 \times 10^{14}\ /\text{cm}^3$

Now, conductivity, $\sigma = n\, e\, \mu_e = 4.41 \times 10^{14} \times 1.6 \times 10^{-19} \times 3800 = 0.268\ /\Omega\,\text{cm}$